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DEPARTMENT: MECHATRONICS

MATRIC NO: 19/ENG05/1004

Assignment

1. $\left[\frac{(x-\cos x)}{x}\right]_{x \to 0}$.

$$\lim_{x \to 0}\left[\frac{[x-\cos x]}{x}\right]$$

Applying L'hopital Rule.

$$\lim_{x \to 0}\left[\frac{(x-\cos x)}{x}\right] = \lim_{x \to 0}\left[\frac{1-(-\sin x]}{1}\right]$$

$$= \lim_{x \to 0}\left[\frac{1+\sin x}{1}\right]$$

$$\lim_{x \to 0}\left[\frac{1+\sin(0)}{1}\right]$$

$$\lim_{x \to 0}\left[\frac{1+0}{1}\right]$$

$$= \frac{1}{1} = 1$$

2. $y = -3\tan 7x\, e^{3t}$. find dy/dx.

$u = -3 \quad v = \tan 7x \quad w = e^{3t}$

$\frac{du}{dx} = 0 \quad \frac{dv}{dx} = 7\sec^2 7x \quad \frac{dw}{dt} = 3e^{3t}$

$$\frac{dy}{dt} = y\left[\frac{1}{u}\frac{du}{dt} + \frac{1}{v}\frac{dv}{dt} + \frac{1}{w}\frac{dw}{dt}\right]$$

$$= y\left[\frac{1}{-3}[0] + \frac{1}{\tan 7x}[7\sec^2 7x] + \frac{1}{e^{3t}}[3e^{3t}]\right]$$

$$= y\left[0 + \left[\frac{7\sec^2 7x}{\tan 7x}\right] + 3\right]$$

$$= y\left[0 + \frac{7\sec^2 7x}{\tan 7x} + 3\right]$$

$$= -3\tan 7x\, e^{3t}\left[\frac{7\sec^2 7x}{\tan 7x}\right] + 3$$

3. $\cos 3x = y$.

$y + \Delta y = \cos 3[x + \Delta x]$

$y + \Delta y = \cos(3x + 3\Delta x)$

$\Delta y = \cos 3[x + \Delta x] - \cos 3x \quad ---[i]$

Recall: $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2} \quad ---(ii)$

comparing equation (ii)

$A = 3x + 3\Delta x \qquad B = 3x$

$\frac{A+B}{2} = \frac{3x + 3\Delta x + 3x}{2} = \frac{6x + 3\Delta x}{2}$

$\frac{A+B}{2} = 3x + 3\Delta x$

Also,

$\frac{A-B}{2} = \frac{3x + 3\Delta x - 3x}{2} = \frac{3\Delta x}{2}$

Hence,

$\Delta y = -2\sin(3x + 3\Delta x)\sin\left[\frac{3\Delta x}{2}\right]$

$\frac{\Delta y}{\Delta x} = \frac{-2\sin(3x + 3\Delta x)\sin\left(\frac{3\Delta x}{2}\right)}{\Delta x}$

$\frac{\Delta y}{\Delta x} = \frac{-2\sin(3x + 3\Delta x)\sin\left[\frac{3\Delta x}{2}\right] \times \frac{1}{2}}{\Delta x \times \frac{1}{2}}$

$\frac{\Delta y}{\Delta x} = \frac{-\sin(3x + 3\Delta x)\sin\left[\frac{3\Delta x}{2}\right]}{\frac{\Delta x}{2}}$

Taking lim as $\Delta x \to 0$.

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = -\sin(3x + 3\Delta x)$

$= -\sin 3x$

$= -\sin 3x$

$\lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x} = \sin\left(\frac{3\Delta x}{2}\right)$

$= \sin(0) = 1$

$\frac{dy}{dx} = -3\sin 3x$

4. $f(x) = 2x^3 - 7x$

$g(x) = -3x$

$f - g(5)$.

$f - g = 2x^3 - 7x - (-3x)$

$= 2x^3 - 7x + 3x$

$= 2x^3 - 4x$

$f - g(5) = 2(5)^3 - 4(5)$

$= 250 - 20$

$= \underline{230}$

2. $f(x) = 4x^2 + 2$

$g(x) = 2x + 3$

$f \circ g(x) = 4(2x + 3)^2 + 2$

$= 4(2x + 3)(2x + 3) + 2$

$= 4(4x^2 + 6x + 6x + 9) + 2$

$= 4(4x^2 + 12x + 9) + 2$

$= 16x^2 + 48x + 36 + 2$

$= 16x^2 + 48x + 38$

6. $x^2 + 2xy + y^2 = 1020.$

$$\frac{d(x^2)}{dx} + 2\left[x\frac{dy}{dx} + y\frac{dx}{dx}\right] + \frac{d(y^2)}{dx} = \frac{d}{dx}(1020)$$

$$2x\frac{dx}{dx} + 2\left[x\frac{dy}{dx} + y\frac{dx}{dx}\right] + 2y\frac{dy}{dx} = 0.$$

$$2x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 0.$$

$$\frac{dy}{dx}[2x + 2y] + 2x + 2y = 0.$$

$$\frac{dy}{dx} = \frac{-[2x + 2y]}{2x + 2y} = -\frac{2(x+y)}{2[x+y]} = -1$$

$$\text{or} = \frac{-2x - 2y}{2x + 2y}$$

7. $y = x^2 \cos x$

$\frac{dy}{dx} = f'(x)$

$u = x^2 \quad v = \cos x$

$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = -\sin x$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x^2(-\sin x) + \cos x(2x)$

$= -x^2(\sin x) + 2x(\cos x) = 2x(\cos x) - x^2(\sin x)$