

1. $y = 2x^2$ at point $(1, 2)$

$$x_1 = 1$$

$$y_1 = 2$$

$$\frac{dy}{dx} = 4x$$

$$\frac{dy}{dx} \Big|_{x=1} = 4 \quad \therefore m = 4$$

For equation of tangent

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 4(x - 1)$$

$$y - 2 = 4x - 4$$

$$y - 4x - 2 + 4 = 0$$

$$y - 4x + 2 = 0 \quad (\text{Equation of the tangent})$$

Equation of normal.

$$m_1 \cdot m_2 = -1$$

$$4m_2 = -1$$

$$m_2 = -1/4$$

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - 2 = -1/4(x - 1)$$

$$4y - 8 = -x + 1$$

$$4y + x - 8 - 1 = 0$$

$$4y + x - 9 = 0 \quad (\text{Equation of the normal})$$

2. $y = 3x^2 - 2x$ at the point $(2, 8)$

$$x_1 = 2$$

$$y_1 = 8$$

$$\frac{dy}{dx} = (6x - 2)$$

$$\left. \frac{dy}{dx} \right|_{x=2} = 10 \quad \therefore m = 10$$

For the equation of tangent,

$$y - y_1 = m(x - x_1)$$

$$y - 8 = 10(x - 2)$$

$$y - 8 = 10x - 20$$

$$y - 10x - 8 + 20 = 0$$

$$y - 10x + 12 = 0 \quad (\text{for the equation of tangent})$$

$$y - y_1 = -1/m(x - x_1)$$

$$y - 8 = -1/10(x - 2)$$

$$10y - 80 = -x + 2$$

$$10y + x - 80 - 2 = 0$$

$$10y + x - 82 = 0 \quad (\text{for the equation of the normal})$$

3. $y = \frac{x^3}{2}$ at the point $(-1, 1/2)$

$$x_1 = -1 \quad \text{and} \quad y_1 = 1/2$$

Apply quotient rule,

$$\frac{dy}{dx} \Big|_{x_1} = 2x_1$$

$$= 3(-1)^2$$

$$= 3$$

Equation of tangent,

$$y - y_1 = m(x - x_1)$$

$$y - (-\frac{1}{2}) = 3[x - (-1)]$$

$$y + \frac{1}{2} = 3(x+1)$$

$$\frac{2y+1}{2} = 3(x+1)$$

$$2y+1 = 6(x+1)$$

$$2y+1 = 6x+6$$

$$2y - 6x - 5 = 0$$

$$6x - 2y + 5 = 0 \quad \left[\text{Equation of } \begin{matrix} \text{tangent} \\ \text{normal} \end{matrix} \right]$$

Equation of normal.

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - (-\frac{1}{2}) = -\frac{1}{3}(x+1)$$

$$y + \frac{1}{2} = -\frac{1}{3}(x+1)$$

$$\frac{2y+1}{2} = -\frac{1}{3}(x+1)$$

$$6y+3 = -2x-2$$

$$6y + 2x + 5 = 0 \quad (\text{Equation of normal})$$

4) $y = 1 + x - x^2$ at point $(-2, 5)$

Solution

$x_1 = -2$ and $y_1 = 5$

$\frac{dy}{dx} = 1 - 2x$

$\frac{dy}{dx} \bigg|_{x=-2} = -2$

$1 - 2(-2) = 1 + 4 = 5 \therefore m = 5$

$y - y_1 = m(x - x_1)$

$y - (-5) = 5(x - (-2))$

$y + 5 = 5(x + 2)$

$y + 5 = 5x + 10$

$y - 5x - 10 + 5 = 0$

$y - 5x - 5 = 0$ (Eqn for tangent)

EQUATION FOR NORMAL.

$y - y_1 = \frac{-1}{m}(x - x_1)$

$y - (-5) = \frac{-1}{5}(x - (-2))$

$y + 5 = \frac{-1}{5}(x + 2)$

Multiply through by 5.

$5y + 25 = -1(x + 2)$

$5y + 25 = -x - 2$

$5y + x + 2 + 25 = 0$

$5y + x + 27 = 0$ (Equation for normal)

5) $y = \frac{1}{x}$ at point $(3, \frac{1}{3})$

Solution: $x_1 = 3$ $y_1 = \frac{1}{3}$

$\frac{dy}{dx} = -1/x^2$

$\frac{dy}{dx} \bigg|_{x=3} = \frac{-1}{3^2} = \frac{-1}{27} \therefore m = \frac{-1}{27}$

Equation of tangent:

$y - y_1 = m(x - x_1)$

$y - \frac{1}{3} = \frac{-1}{27}(x - 3)$

$27y - 9 = -x + 3$

$27y - 9 + x - 3 = 0$

$27y + x - 9 - 3 = 0$

$27y + x - 12 = 0$ [Equation for tangent]

EQUATION FOR NORMAL.

$y - y_1 = \frac{-1}{m}(x - x_1)$

$y - \frac{1}{3} = -\left(\frac{-1}{27}\right)(x - 3)$

$y - \frac{1}{3} = \frac{1}{27}(x - 3)$

$27y - 9 = x - 3$

$27y - x - 9 + 3 = 0$

$27y - x - 6 = 0$ (Equation for normal)