Maths 104 19/MHS 01/153

Find the Equation of the tangent and find the normal for the following given curves

1 $y = 2x^2$ at $(1, 2)$

$y = 2x^2$

$\frac{dy}{dx} = m = 4x$

At $x = 1$, $m = 4(1) = 4$

Eqn of tangent

$m = m_1$

$m_1 = \frac{y - y_1}{x - x_1}$ $\quad 4 = \frac{y - 2}{x - 1}$

$4(x - 1) = y - 2$

$4x - 4 = y - 2$

$y = 4x - 2$

Eqn of the normal

$m_2 = -1/m$

$m_2 = -1/4$

But $m_2 = \frac{y - y_1}{x_2 - x_1}$; $\frac{-1}{4} = \frac{y - 2}{2 - 1}$

$4(y-2) = -1(x-1)$

$4y - 8 = -x + 1$

$4y = -x + 9$

$y = 3x^2 - 2x$ at $(2, 8)$

$y = 3x^2 - 2x$

$\frac{dy}{dx} = 6x - 2$

Eqn of the tangent

$m_1 = m$ $\qquad m_1 = 10$

$m_1 = \frac{y - y_1}{x - x_1}$

$10 = \frac{y - 8}{x - 2}$

$10(x - 2) = y - 8$

$10x - 20 = y - 8$

$y = 10x - 20 + 8$

$y = 10x - 12$

$4(y-2) = -1(x-1)$

$4y - 8 = -x + 1$

$4y = -x + 9$

$y = 3x^2 - 2x$ at $(2, 8)$

$y = 3x^2 - 2x$

$\frac{dy}{dx} = 6x - 2$

Eqn of the tangent

$m_1 = m$ $\qquad$ $m_1 = 10$

$m_1 = \frac{y - y_1}{x - x_1}$

$10 = \frac{y - 8}{x - 2}$

$10(x - 2) = y - 8$

$10x - 20 = y - 8$

$y = 10x - 20 + 8$

$y = 10x - 12$

$3(x+1) = 2(y+1/2)$

$3x + 3 = 2y + 1$

$2y = 3x + 3 - 1$

$2y = 3x + 2$

Eqn of normal

$m_2 = -1/m \quad \therefore \quad m_2 = -1/3/2 = -2/3$

$m_2 = \frac{y - y_1}{x_2 - x_1}$

$\frac{-2}{3} = \frac{y - (-1/2)}{x - (-1)}$

$-2(x+1) = 3(y+1/2)$

$3y = -2x - 2 - 3/2$

$3y = -2x - 7/2$

$6y = -4x - 7$

4 $y = 1 + x - x^2$ at $(-2, -5)$

$y - 1 + x - x^2$

$\frac{dy}{dx} = m = 1 - 2x$

At $x = -2 \quad m = 1 - 2(-2)$

$m = 1 + 4 = 5$

Eqn of the normal

$m_2 = -1/m \quad \therefore \quad m_2 = -1/10$

$m_2 = \frac{y - y_1}{x - x_1} \quad \therefore \quad -1/10 = \frac{y - 8}{x - 2}$

$10(y - 8) = -1(x - 2)$

$10y - 80 = -x + 2$

$10y = -x + 2 + 80$

$10y = -x + 82$

3. $y = x^{3/2}$ at $(-1, -1/2)$

$y = x^{3/2} = 1/2(x^3)$

$m = \frac{dy}{dx} = \frac{1}{2}(3x^2) = \frac{3x^2}{2}$

At $x = -1, \quad m = \frac{3}{2}(-1)^2$

$m = \frac{3}{2}$

Eqn of the tangent

$m_1 = m \qquad m_1 = 3/2$

$m_1 = \frac{y - y_1}{x - x_1} \quad \therefore \quad \frac{3}{2} = \frac{y - (-1/2)}{x - (-1)}$

5. $y = 1/x$ at $(3, 1/3)$

$$y = 1/x = x^{-1}$$

$$\frac{dy}{dx} = m = -1(x^{-1-1}) = -1(x^{-2}) = -1/x^2$$

At $x = 3$, $m = -1/3^2 = -1/9$

Eqn of the tangent

$m_1 = m$ $\quad m_1 = -1/9$

$$m_1 = \frac{y - y_1}{x - x_1} \quad \therefore \frac{-1}{9} = \frac{y - 1/3}{x - 3}$$

$$9(y - 1/3) = -1(x - 3)$$

$$9y - 3 = -x + 3$$

$$9y = -x + 3 + 3$$

$$9y = -x + 6$$

Eqn of normal

$$m_2 = -1/m$$

$$m_2 = \frac{y - y_1}{x - x_1} \qquad m_2 \; -1/-1/9 = 9$$

$$9 = \frac{y - 1/3}{x - 3}$$

$$9(x - 3) = y - 1/3$$

$$9x - 27 = y - 1/3$$

$$y = 9x - 27 + 1/3$$

$$3y = 27x - 81 + 1$$

Eqn of the tangent

$m_1 = m \quad \therefore m_1 = 5$

$m_1 = \frac{y - y_1}{x - (-2)}$

$5 = \frac{y - (-5)}{x - (-2)} \quad ; \quad 5(x+2) = y+5$

$5x + 10 = y + 5$

$y = 5x + 10 - 5$

$y = 5x + 5$

Eqn of the normal

$m_2 = -1/m \quad m_2 = -1/5$

$m_2 = \frac{y - y_1}{x - x_1} = \frac{-1}{5} = \frac{y-(-5)}{x-(-2)}$

$-1(x+2) = 5(y+5)$

$-x - 2 = 5y + 25$

$5y = x - 2 - 25$

$5y = x - 27$