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MATH104 ASSIGNMENT

MBBS

19/MHS01/192

Equation of the tangent and normal.

For the curves in problem 1 to 5, at the points given find

- a) the equation of the tangent
- b) the equation of the normal.

solution.

1. $y = 2x^2$ at point $(1, 2)$

$x_1 = 1, y_1 = 2$

$y = 2x^2$

$\frac{dy}{dx} = 4x$

$= 4(1)$

$\frac{dy}{dx} = 4$
 $\frac{dy}{dx} = m_1 \therefore m_1 = 4$

a) $x_1 = 1, y_1 = 2, m_1 = 4$

$y - y_1 = m(x - x_1)$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y - 2 + 4 - 4x = 0$

$y - 4x + 2 = 0$ → equation of the tangent.

b) $y - y_1 = m_2(x - x_1) \quad m_2 = -\frac{1}{m_1}$

$y - y_1 = -\frac{1}{m_1}(x - x_1)$

$4x(y - 2) = -\frac{1}{4}(x - 1) \times 4$

$4y - 8 = -x + 1$

$4y + x - 8 - 1 = 0$

$4y + x - 9 = 0$ → equation of the normal

(2)

2) $y = 3x^2 - 2x$ at point $(2, 8)$
~~at~~ $x_1 = 2$ $y_1 = 8$

$$\frac{dy}{dx} = 6x - 2$$

$$\frac{dy}{dx} = 6(2) - 2$$

$$\frac{dy}{dx} = 12 - 2$$

$$\frac{dy}{dx} = 10 = \text{slope}$$

$$\frac{dy}{dx} = m_1$$

$$\therefore m_1 = 10$$

a) $x_1 = 2, y_1 = 8, m_1 = 10$

$$y - y_1 = m_1(x - x_1)$$

$$y - 8 = 10(x - 2)$$

$$y - 8 = 10x - 20$$

$$y - 8 - 10x + 20 = 0$$

$$y - 10x + 12 = 0 \rightarrow \text{equation of the tangent}$$

b) $y - y_1 = \frac{-1}{m_1}(x - x_1)$ $m_2 = \frac{-1}{m_1}$

$$10x(y - 8) = \frac{-1}{10}(x - 2) \times 10$$

$$10y - 80 = -x + 2$$

$$10y - 80 + x - 2 = 0$$

$$10y + x - 82 = 0 \rightarrow \text{equation of the normal}$$

3) $y = \frac{x^3}{2}$ at point $(-1, -\frac{1}{2})$

$$\frac{dy}{dx} = \frac{3x^2}{2}$$

$$\frac{dy}{dx} \Big|_{(-1, -\frac{1}{2})} = \frac{3 \times (-1)^2}{2}$$

$$\frac{dy}{dx} = \frac{3}{2}$$

$$\frac{dy}{dx} = m_1 \quad \therefore m_1 = \frac{3}{2}$$

(3)

a $x_1 = -1, y = -1/2, m_1 = 3$
 $y - y_1 = m_1(x - x_1)$
 $y - (-1/2) = \frac{3}{2}(x + 1)$

$$y + 1/2 = \frac{3}{2}(x + 1)$$

$$\frac{2y + 1}{2} \times \frac{2}{2} = \frac{3x + 3}{2}$$

$$2(2y + 1) = 2(3x + 3)$$

$$4y + 2 = 6x + 6$$

$$4y - 6x + 2 - 6 = 0$$

$$\frac{4y}{2} - \frac{6x}{2} - \frac{4}{2} = \frac{0}{2} \quad +$$

$$2y - 3x - 2 = 0 \rightarrow \text{equation of the tangent}$$

$$m_2 = -\frac{1}{m_1}$$

b $y - y_1 = -\frac{1}{m_1}(x - x_1)$

$$y - (-1/2) = -\frac{1}{3/2}(x - (-1))$$

$$6x(y + 1/2) = -\frac{2}{3}(x + 1) \times 6$$

$$6y + 3 = -4(x + 1)$$

$$6y + 3 = -4x - 4$$

$$6y + 3 - 4x + 4 = 0$$

$$6y - 4x + 7 = 0 \rightarrow \text{equation of the normal.}$$

4 $y = 1 + x - x^2$ at point $(-2, 5)$

$$\frac{dy}{dx} = 1 - 2x$$

$$\frac{dy}{dx} \Big|_{(-2, 5)} = 1 - 2(-2)$$

$$\frac{dy}{dx} \Big|_{(-2, 5)} = 1 + 4$$

$$\frac{dy}{dx} = 5$$

$$\frac{dy}{dx} = m_1 \therefore m_1 = 5$$

(4)

$$\begin{aligned}x_1 &= -2, y_1 = -5, m_1 = 5 \\y - y_1 &= m_1(x - x_1) \\y + 5 &= 5(x + 2) \\y + 5 &= 5x + 10 \\y + 5 - 5x - 10 &= 0 \\y - 5x - 5 &= 0 \rightarrow \text{equation of the tangent} \\m_2 &= -\frac{1}{m_1}\end{aligned}$$

$$y - y_1 = -\frac{1}{m_1}(x - x_1)$$

$$y + 5 = -\frac{1}{5}(x + 2)$$

$$5(y + 5) = -1(x + 2) \quad \times \cancel{5}$$

$$5y + 25 = -x - 2$$

$$5y + 25 + x + 2 = 0$$

$$5y + x + 27 = 0 \rightarrow \text{equation of the normal}$$

(5) $y = \frac{1}{x}$ at point $(3, \frac{1}{3})$

$$\frac{dy}{dx} = x^{-2}$$

$$\frac{dy}{dx} \Big|_{(3, \frac{1}{3})} = \left(\frac{1}{3}\right)^{-2} = \frac{1}{3^2}$$

$$\frac{dy}{dx} = \frac{1}{9}$$

$$\frac{dy}{dx} = m_1, m_1 = \frac{1}{9}$$

$$x_1 = 3, y_1 = \frac{1}{3}, m_1 = \frac{1}{9}$$

$$y - y_1 = m_1(x - x_1)$$

$$y - \frac{1}{3} = \frac{1}{9}(x - 3)$$

$$9 \times (y - \frac{1}{3}) = \frac{1}{9}(x - 3) \times 9$$

(5)

$$9y - 8 = x - 3$$

$$9y - 3 - x + 3 = 0$$

$$9y - x = 0 \rightarrow \text{equation of the normal.}$$