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1. $y = 2x^2$ at the point $(1,2)$.
Solution

$y = 2x^2$ $\quad dy/dx = 4x$.

$\left.\frac{dy}{dx}\right|_{x=1} = 4(1) = 4$. $M_1 = 4$.

$y - y_1 = m(x - x_1)$ $\quad ; y_1 = 2 ; x_1 = 1$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y - 2 + 4 - 4x = 0$

$y + 2 - 4x = 0$ which gives the equation of the tangent.

Equation of the normal.

$M_1 M_2 = -1 ; M_1 = 4$

$M_2 = \frac{-1}{M_1} = -\frac{1}{4}$ $\quad ; y_1 = 2 ; x_1 = 1$

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{4}(x - 1)$

$4y - 8 = -x + 1$

$4y - 9 + x = 0$ which gives the equation of the normal.

2. $y = 3x^2 - 2x$ at the point $(2,8)$.
Solution

$y = 3x^2 - 2x$ $\quad dy/dx = 6x - 2$

$\left.\frac{dy}{dx}\right|_{x=2} = 6(2) - 2 = 12 - 2 = 10$.

$M_1 = 10$. $\quad y_1 = 8 ; x_1 = 2$

$y - y_1 = m(x - x_1)$

$y - 8 = 10(x-2)$

$y - 8 = 10x - 20$

$y - 8 - 10x + 20 = 0$

$y + 12 - 10x = 0$ which gives the equation of the tangent.

Equation of the normal.

$M_1 M_2 = -1$ $\quad M_2 = -1/M_1 = -1/10$.

$y - y_1 = m(x - x_1)$

$y - 8 = \frac{-1}{10}(x-2)$

$10(y-8) = -1(x-2)$

$10y - 80 = -x + 2$

$10y - 80 + x - 2 = 0$

$10y - 82 + x = 0$ which gives the equation of the normal.

3) $y = \frac{x^3}{2}$ at the point $(-1, -1/2)$

Solution

$y = \frac{x^3}{2}$

$y = u/v$. Using quotient rule

$dy/dx = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$u = x^3$; $du/dx = 3x^2$

$v = 2$; $dv/dx = 0$

Substituting:

$\frac{dy}{dx} = \frac{2(3x^2) - (x^3)(0)}{(2)^2}$

$\frac{dy}{dx} = \frac{6x^2 - 0}{4} = \frac{6x^2}{4} = \frac{3x^2}{2}$.

$\left.\frac{dy}{dx}\right|_{x=-1}$ $\quad \frac{3(-1)^2}{2} = \frac{3(1)}{2} = \frac{3}{2}$

$M_1 = 3/2$.

$y-y_1=m(x-x_1)$; $y_1=-\frac{1}{2}$ $x_1=-1$

$y-(-\frac{1}{2})=\frac{3}{2}(x-(-1))$

$\frac{y}{1}+\frac{1}{2}=\frac{3x+3}{2}$

$\frac{2y+1}{2}=\frac{3x+3}{2}$

$\cancel{2}(2y+1)=\cancel{2}(3x+3)$

$2y+1-3x-3=0$

$2y-2-3x=0$ which gives the equation of the tangent.

Equation of the normal.

$M_1M_2=-1$ $M_2=-\frac{1}{M_1}$ $=-\frac{1}{3/2}$

$\therefore M_2=-\frac{2}{3}$

$y-y_1=m(x-x_1)$

$y-(-\frac{1}{2})=-\frac{2}{3}(x-(-1))$

$\frac{y}{1}+\frac{1}{2}=\frac{-2x-2}{3}$

$\frac{2y+1}{\cancel{2}}=\frac{-\cancel{2}(x+1)}{3}$

$3(2y+1)=-(x+1)$

$6y+3=-x-1$

$6y+3+x+1=0$

$6y+4+x=0$ which gives the equation of the normal.

4) $y=1+x-x^2$ at the point $(-2,-5)$

$\frac{dy}{dx}=1-2x$; $\left.\frac{dy}{dx}\right|_{x=-2}=1-2(-2)=1+4=5$

$\therefore M_1=5$

$y-y_1=m(x-x_1)$ $y_1=-5$; $x_1=-2$

$y-(-5)=5(x-(-2))$

$y+5=5x+10$

$y+5=5x+10$

$y+5-5x-10=0$

$y-5x-5=0$ which gives the equation of the tangent.

Equation of the normal

$M_1M_2=-1$

$M_2=\frac{-1}{5}$

$y-y_1=m(x-x_1)$

$y-(-5)=\frac{-1}{5}(x-(-2))$

$y+5=\frac{-1}{5}(x+2)$

$5(y+5)=-1(x+2)$

$5y+25=-x-2$

$5y+25+x+2=0$

$5y+27+x=0$ which gives the equation of the normal.

5. $y=1/x$ at the point $(3, 1/3)$

Solution

$y=1/x \quad = 1x^{-1}$

$dy/dx=-1x^{-2} \quad = -1/x^2$

$\left.\frac{dy}{dx}\right|_{x=3} = \frac{-1}{3^2} = \frac{-1}{9} = M_1$

$y-y_1=m(x-x_1)$

$y_1=1/3 \; ; \; x_1=3$

$y-\frac{1}{3}=\frac{-1}{9}(x-3)$

$\frac{3y-1}{3}=\frac{-x+3}{9}$

$\cancel{9}(3y-1)=\cancel{3}(-x+3)$

$3(3y-1)=-x+3$

$9y-3+x-3=0$

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$9y-6+x=0$ which gives the equation of the tangent.

Equation of the normal.

$M_1M_2=-1$

$M_2=-1/M_1 = -1/-1/9 = 9$

$y-y_1=m(x-x_1)$

$y-\frac{1}{3}=9(x-3)$

$\frac{3y-1}{\cancel{3}_1}=\overset{3}{\cancel{9}}(x-3)$

$3y-1=3x-9$

$3y-1-3x+9=0$

$3y-3x+8=0$ which gives the equation of the normal.