(143)

MAT 100 11/04/2020

① $y = \dfrac{(x+1)^2(x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}$

$\ln y = \left[\ln(x+1)^2 + \ln(x-2)^{1/2}\right] - \left[\ln(2x-1) + \ln(x-3)^{4/3}\right]$

$\dfrac{1}{y}\dfrac{dy}{dx} = \left[\dfrac{1}{(x+1)^2} \cdot 2(x+1) + \dfrac{1}{(x-2)^{1/2}} \cdot \dfrac{(x-2)^{-1/2}}{2}\right] - \left[\dfrac{1}{2x-1} \cdot 2 + \dfrac{1}{(x-3)^{4/3}} \cdot \dfrac{4}{3}(x-3)^{1/3}\right]$

$\dfrac{1}{y}\dfrac{dy}{dx} = \left[\dfrac{2(x+1)}{(x+1)^2} + \dfrac{\frac{1}{\sqrt{x-2}}}{2\sqrt{x-2}}\right] - \left[\dfrac{2}{(2x-1)} + \dfrac{4(x-3)^{1/3}}{3(x-3)^{4/3}}\right]$

$\dfrac{1}{y}\dfrac{dy}{dx} = \left[\dfrac{2}{(x+1)} + \dfrac{1}{2(x-2)}\right] - \left[\dfrac{2}{(2x-1)} + \dfrac{4}{3(x-3)}\right]$

$\dfrac{dy}{dx} = y\left[\dfrac{2}{(x+1)} + \dfrac{1}{2(x-2)} - \dfrac{2}{(2x-1)} - \dfrac{4}{3(x-3)}\right]$

$\dfrac{dy}{dx} = \dfrac{(x+1)^2(x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}\left[\dfrac{2}{(x+1)} + \dfrac{1}{(2x-4)} - \dfrac{2}{(2x-1)} - \dfrac{4}{3x-4}\right]$

② $y = \dfrac{3e^{x}\sin 2x}{x^{5/2}}$

$\ln y = \ln(3e^{x}) + \ln(\sin 2x) - \ln(x^{5/2})$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{3e^{x}} \cdot 3e^{x} + \dfrac{1}{\sin 2x} \cdot 2\cos 2x - \dfrac{1}{x^{5/2}} \cdot \dfrac{5}{2}x^{3/2}$

$\dfrac{1}{y} \cdot \dfrac{dy}{dx} = 1 + \dfrac{2\cos 2x}{\sin 2x} - \dfrac{5}{2}x^{3/2 - 5/2}$

$\dfrac{dy}{dx} = y\left[1 + \dfrac{2\cos 2x}{\sin 2x} - \dfrac{5x^{-1}}{2}\right]$

$$\frac{dy}{dx} = \frac{3e^{x} \sin 2x}{x^{3/2}} \left[1 + \frac{2\cos 2x}{\sin 2x} - \frac{3x^{-1}}{2} \right]$$

③ $\int 4\sec^2(3m+1)\,dm$

$4\int \sec^2(3m+1)\,dm$

$u = 3m+1$

$du = 3dm \therefore dm = \frac{du}{3}$

$4\int \sec^2(u) \frac{du}{3}$

$\frac{4}{3}\int \sec^2(u)\,du$

$\frac{4}{3} \tan u + C$

$= \frac{4}{3} \tan(3m+1) + C$

④ $\int 2t\,(3t^2-1)^{1/2}\,dt$

$u = \sqrt{3t^2-1}$

$u^2 = 3t^2 - 1$

$3t^2 = u^2 + 1$

$t^2 = \frac{u^2+1}{3}$

$\frac{dt}{du} = \frac{1}{2}\left(\frac{u^2+1}{3}\right)^{-1/2} \cdot \frac{2u}{3}$

$\frac{dt}{du} = \frac{u}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$dt = \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$$\int 2\left(\frac{u^2+1}{3}\right)^{1/2} \cdot u \cdot \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$$

$$= \frac{2}{3}\int u^2\left(\frac{u^2+1}{3}\right)^{1/2 - 1/2} du$$

$$= \frac{2}{3}\int u^2\, du$$

$$= \frac{2}{3}\left[\frac{u^3}{3}\right] + C$$

$$= \frac{2u^3}{9} + C$$

$$= \frac{2(3t^2-1)^{3/2}}{9} + C$$

(5) $\int \frac{2x}{\sqrt{4x^2-1}}\, dx$

$$u = \sqrt{4x^2-1}$$

$$u^2 = 4x^2 - 1$$

$$4x^2 = u^2+1$$

$$x^2 = \frac{u^2+1}{4}$$

$$x = \sqrt{\frac{u^2+1}{4}}$$

$$\frac{dx}{du} = \frac{1}{2}\left(\frac{u^2+1}{4}\right)^{-1/2} \cdot \frac{u}{2}$$

$$\frac{dx}{du} = \frac{u}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$$

$$dx = \frac{u\,du}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$$

$$\int \frac{x\left(\frac{u^2+1}{4}\right)^{1/2}}{4} \cdot \frac{u\,du}{4/2}\left(\frac{u^2+1}{4}\right)^{-1/2}$$

$$\frac{1}{2}\int \left(\frac{u^2+1}{4}\right)^{1/2 - 1/2} du$$

$$\frac{1}{2}\int du$$

$$= \frac{u}{2} + C$$

$$= \frac{\sqrt{4x^2-1}}{2} + C$$