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MAT 104
19/MHS01/018
MHS/MBBS

For the curve from equation 1 to 5, at the points given
Equation of the tangent
Equation of the Normal

$y = 2x^2$ at the point $(1, 2)$

$y = 3x^2 - 2x$ at the point $(2, 8)$

$y = x^{3/2}$ at the point $(-1, -1/2)$

$y = 1 + x - x^2$ at the point $(-2, 5)$

$y = 1/x$ at the point $(3, 1/3)$

Solution

$y = 2x^2$; Points $(1, 2)$

$\frac{dy}{dx} = 4x$

Gradient at $x = 1$

$\frac{dy}{dx}\Big|_{x=1} \quad 4(1) = 4 \quad m = 4$

$x = 1, \; y = 2$

Equation of a Tangent

$y - y_1 = m(x - x_1)$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y - 4x + 4 - 2 = 0$

$y - 4x + 2 = 0$

Equation of the tangent $= y - 4x + 2 = 0$

Equation of Normal

$M_1 \cdot M_2 = -1$

$4 M_2 = -1$

$M_2 = -1/4$

$y - y_1 = M_2(x - x_1)$

$y - 2 = -1/4(x - x_1)$

$\frac{y - 2}{1} = \frac{-x + 1}{4}$

$4y - 8 = -x + 1$

$4y + x - 9 = 0$

Equation of the Normal

$= 4y + x - 9 = 0$

1

iii] $y = 3x^2 - 2x$ at the point $(2, 8)$

$\frac{dy}{dx} = 3x^2 - 2x = 6x - 2$

Find the gradient at $x = 2$

$\frac{dy}{dx} / x = 2$

$m = 6x - 2$

$m = 6(2) - 2$

$m = 12 - 2 = 10$

Equation of a tangent

$y - y_1 = m(x - x_1)$

$y - 8 = 10(x - 2)$

$y - 8 = 10x - 20$

$y - 10x - 8 + 20 = 0$

$y + 24 - 10x = 0$ or $y - 10x + 24 = 0$

Equation of the tangent $\Rightarrow y - 10x + 24 = 0$

Equation of the Normal

$M_1 M_2 = -1$

$10 M_2 = -1$

$M_2 = -\frac{1}{10}$

$y - y_1 = M(x - x_1)$

$y - 8 = -\frac{1}{10}(x - 2)$

$10y - 80 = -x + 2$

$10y + x - 80 = 0$

Equation of the Normal $\Rightarrow 10y + x - 80 = 0$

(2)

iii. 1 $y = x^3$ at the point $(-1, -\frac{1}{2})$

$\frac{dy}{dx} = 3x^2$

Find the gradient at -1

$\frac{dy}{dx}|_{x=-1}$

$m = 3x^2$

$m = 3(-1)^2 = 3$

Equation of a tangent

$y - y_1 = m(x - x_1)$

$y + \frac{1}{2} = 3(x - -1)$

$y + \frac{1}{2} = 3x + 3$

$y + \frac{1}{2} - 3 - 3x = 0$

$y - 3x - 2\frac{1}{2} = 0$

Equation of the tangent $\Rightarrow y - 3x - 2\frac{1}{2} = 0$

Equation of the Normal

$m_1 \cdot m_2 = -1$

$3 \cdot m_2 = -1$

$m_2 = -\frac{1}{3}$

$y - y_1 = m(x - x_1)$

$y + \frac{1}{2} = -\frac{1}{3}(x + 1)$

$y + \frac{1}{2} = -\frac{1}{3}x - \frac{1}{3}$

$y + \frac{1}{2} + \frac{1}{3} + \frac{1}{3}x = 0$

$y + \frac{1}{3}x + \frac{5}{6} = 0$

Equation of the Normal $\Rightarrow y + \frac{1}{3}x + \frac{5}{6} = 0$

(3)

(v.) $y = 1 + 2x - x^2$ at the point $(-2, 5)$

$\frac{dy}{dx} = 1 - 2x$

Gradient at $x = -2$

$\frac{dy}{dx}\Big|_{x=-2}$

$m = 1 - 2x$

$m = 1 - 2(-2)$

$m = 1 + 4 = 5$

Equation of a tangent

$y - y_1 = m(x - x_1)$

$y - 5 = 5(x + 2)$

$y - 5 = 5x + 10$

$y - 5x - 5 - 10 = 0$

$y - 5x - 15 = 0$

Equation of a tangent $= y - 5x - 15 = 0$

4

Equation of the Normal

$m_1 \cdot m_2 = -1$

$5 m_2 = -1$

$m_2 = -1/5$

$y - y_1 = m(x - x_1)$

$y - 5 = -1/5(x + 2)$

$y - 5 = -1/5 x - \frac{2}{5}$

$y + 1/5 x - 5 + \frac{2}{5} = 0$

$y + \frac{1}{5}x - \frac{23}{5} = 0$

Equation of the Normal: $y + 1/5 x - 23/5 = 0$

8) $y = 1/x$ at the point $(3, 1/3)$

$\frac{dy}{dx} = x^{-1} = -1x^{-2}$

Find the gradient at $x = 3$

$\frac{dy}{dx}\Big|_{x=3}$ $\quad m = -1 \times 3^{-2}$

$m = -1 \times \frac{1}{3^2}$

$m = -1 \times \frac{1}{9}$

$m = -1/9$

Equation of the tangent

$y - y_1 = m(x - x_1)$

$y - 1/3 = -1/9(x - 3)$

$9y - 9/3 = -x + 3$

$9y - 3 = -x + 3$

$9y + x = 0$

Equation of the tangent $\Rightarrow 9y + x = 0$

Equation of the Normal

$m_1 m_2 = -1$

$-1/9 \cdot m_2 = -1$

$m_2 = -1/-1/9$

$m_2 = -1 \times \frac{-9}{1}$

$m_2 = 9$

$y_2 - y_1 = m(x - x_1)$

$y - \frac{1}{3} = 9(x - 3)$

$y - 1/3 = 9x - 27$

$y - 9x - 1/3 + 27 = 0$

$y - 9x + 26\,7 = 0$

Equation of the Normal $\Rightarrow y - 9x + 26\,7 = 0$