1) $\frac{x - \cos x}{x}$

$\frac{dy}{dx} = \frac{1 + \sin x}{1} = \frac{1+0}{1}$ as $x \to 0$

$\frac{dy}{dx} = 1$

2) $y = -3 \tan 7x \, e^{3x}$

Let $u = -3 \tan 7x$ and $v = e^{3x}$

$\frac{du}{dx} = -21 \sec^2 7x$ $\quad \frac{dv}{dx} = 3e^{3x}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= -3 \tan 7x (3e^{3x}) + e^{3x}(-21 \sec^2 x)$

3) $y = \cos 3x$

by differentiating using first principle

$y + \Delta y = \cos(3x + \Delta x)$

$\Delta y = \cos(3x + \Delta x) - y$

$\Delta y = \cos(3x + \Delta x) - \cos 3x$

Recall $\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$

$A = 3x + \Delta x$, $B = 3x$

$\frac{A+B}{2} = \frac{3x + \Delta x + 3x}{2} = \frac{6x + \Delta x}{2} = 3x + \frac{\Delta x}{2}$

$\frac{A-B}{2} = \frac{3x + \Delta x - 3x}{2} = \frac{\Delta x}{2}$

$\frac{\Delta y}{\Delta x} = \frac{-2 \sin\left(3x + \frac{\Delta x}{2}\right) \cdot \sin\left(\frac{\Delta x}{2}\right) \times 3/2}{\Delta x \times 3/2}$

$\frac{\Delta y}{\Delta x} = -3 \sin(3x + 0) \cdot \lim_{\Delta x \to 0} \frac{\sin\left(\frac{3\Delta x}{2}\right)}{\left(\frac{3\Delta x}{2}\right)}$

since $\lim_{\Delta x \to 0} \sin \left(\frac{3\Delta x}{2}\right) / \left(\frac{3\Delta x}{2}\right) = 1$

$\frac{\Delta y}{\Delta x} = -3 \sin 3x$

4) $f(x) = 2x^3 - 7x \qquad g(x) = -3x$

$(f-g)(5) = f(5) - g(5)$

$= [2(5)^3 - 7(5)] - [-3(5)]$

$= (250 - 35) - (-15)$

$= 215 + 15$

$= 230$

5) $fog(x) = ?$ $\quad f(x) = 4x^2 + 2$ $\quad g(x) = 2x + 3$

$fog(x) = 4(2x+3)^2 + 3$

$fog(x) = 4(4x^2 + 12x + 9) + 3$

$fog(x) = 16x^2 + 48x + 36 + 3$

$fog(x) = 16x^2 + 48x + 39$

6) $x^2 + 2xy + y^2 = 1020$

Slope (gradient) $= M = \frac{dy}{dx}$

$2x\frac{dx}{dx} + 2x\frac{dy}{dx} + 2y\frac{dx}{dx} + 2y\frac{dy}{dx} = 0$

$2x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 0$

$2x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 2y$

$\frac{dy}{dx}(2x + 2y) = -2x - 2y$

$\frac{dy}{dx} = \frac{-2x - 2y}{2x + 2y}$

$= \frac{-2(x+y)}{x(x+y)}$

$= -1 \quad (1) \quad = -1$

7) $y = x^2 \cos x$

let $u = x^2 \quad v = \cos x$

$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = -\sin x$

$\frac{dy}{dx} = U\frac{dv}{dx} + V\frac{du}{dx}$

$\frac{dy}{dx} = x^2(-\sin x) + \cos x(2x)$

$= -x^2 \sin x + 2x\cos x$