(5)

$y = 1/x$ at the point $(3, 1/3)$

Soln

$$\frac{dy}{dx} = 1x^{-2}$$

$$-(3)^{-2} = 1/9$$

$$\frac{-1}{9} = \frac{y - y_1}{x - x_1}$$

$$\frac{-1}{9} = \frac{y - 1/3}{x - 3}$$

$$-1(x-3) = 9(y - 1/3)$$

$$-x + 3 = 9y - 3$$

∴ The equation of the gradient $= 9y + x - 9 = 0$

The equation of the normal

$$\frac{dy}{dx} = -x^{-2}$$

$$-(3)^{-2} = -1/9$$

$$-1/9 = -1$$

$$m = 9$$

$$9 = \frac{y - 1/3}{x - 3}$$

$$9(x-3) = y - 1/3$$

$$9x - 27 = y - 1/3$$

$$9x - y = -1/3 + 27$$

$$9x - y + 82/3 = 0$$

∴ The equation of the normal $= 9x - y + 82/3 = 0$

(4)

$y = 1 + x - x^2$ at the point $(-2, -5)$

$\frac{dy}{dx} = 1 - 2x$

$\frac{dy}{dx} = -2x + 1$

Find the gradient at point $x = -2$

$\left.\frac{dy}{dx}\right|_{x=-2}$ $\quad m = -2(-2) + 1$

$m = 4 + 1$

$m = 5$

Equation of the tangent

$y - y_1 = m(x - x_1)$

$y - (-5) = m(x + 2)$

$y + 5 = 5(x + 2)$

$y + 5 = 5x + 10$

$y - 5x + 5 - 10 = 0$

$y - 5x - 5 = 0$

∴ Equation of the tangent

$= y - 5x - 5 = 0$

Equation of the normal

$m_1 \cdot m_2 = -1$

$5 \cdot m_2 = -1$

$m_2 = -1$

$y - y_1 = m(x - x_1)$

$y - (-5) = -1/5 (x - (-2))$

$y + 5 = -1/5 (x + 2)$

$5y + 25 = -x - 2 = 0$

$5y + x + 25 + 2 = 0$

∴ Equation of the normal is =

$5y + x - 27 = 0$

Jaja Emmanuella Adaugo
MAT 104
191MHS01215
MHS/MBBS

Assignment:

For the curves in problem 1 to 5, at the point given, Find
a) The equation of the tangent
b) The equation of the normal.

(1)

Soln

1. $y = 2x^2$ at the point (1, 2)

Soln

$y = 2x^2$, points (1, 2)

$\frac{dy}{dx} = 4x$

Finding the gradient at $x = 1$

$= \left.\frac{dy}{dx}\right|_{x=1}$

$= 4(1) = 4$

$m = 4$

$x_1 = 1, y_1 = 2$

Equation of a tangent

$y - y_1 = m(x - x_1)$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y - 2x - 4 + 4 = 0$

Equation of the tangent $= y - 2x = 0$

$y - 4x + 4 - 2 = 0$

$y - 4x + 2 = 0$

∴ Equation of the tangent $= y - 4x + 2 = 0$

ii Equation of normal

$m_1 \cdot m_2 = -1$

$4 \cdot m_2 = -1$

$m_2 = -\frac{1}{4}$

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{4}(x - 1)$

$\frac{y-2}{1} = \frac{-x+1}{4}$

Cross multiply

$4y - 8 = -x + 1$

$4y + x - 7 = 0$

∴ Equation of the normal

$= 4y + x - 7 = 0$

(2)

2. $y = 3x^2 - 2x$ at the point (2, 8)

Soln

$\frac{dy}{dx} = 3x^2 - 2x = 9x - 2$

Find the gradient at $x = 2$

$\left.\frac{dy}{dx}\right|_{x=2}$

$m = 9x - 2$

$m = 9(2) - 2$

$m = 18 - 2 = 16$

Equation of a tangent

$y - y_1 = m(x - x_1)$

$y - 8 = 16(x - 2)$

$y - 8 = 16x - 32$

$y - 16x - 8 + 32 = 0$

$y - 16x + 24 = 0$

∴ Equation of the tangent = $y - 16x + 24 = 0$

ii) Equation of the normal

$m_1 \cdot m_2 = -1$

$16 \cdot m_2 = -1$

$m_2 = \frac{-1}{16}$

$y - y_1 = m(x - x_1)$

$y - 8 = -1/16(x - 2)$

$16y - 128 = -x + 2$

$16y + x - 130 = 0$

(3)

$y = x^3/2$ at point $(-1, -1/2)$

$y = \frac{x^3}{2}$

$\frac{dy}{dx} = 2x^2$

Find the gradient at $x = 1$

$m = 2(-1)^2$

$m = 2(1)$

$m = 2$

Equation of the tangent

$y - y_1 = m(x - x_1)$

$y - (-1/2) = m(x - (-1))$

$y + 1/2 = m(x + 1)$

$y + 1/2 = 2(x + 1)$

$y + 1/2 = 2x + 2$

$y - 2x + 1/2 - 2 = 0$

$y - 2x + -3/2 = 0$

$y - 2x - \frac{3}{2} = 0$

Equation of the Normal

$m_1 \cdot m_2 = -1$

$m_2 = \frac{-1}{m_1}$

$m_2 = -1/2$

$y - y_1 = m(x - x_1)$

$y - (-1/2) = -1/2(x - (-1))$

$y + 1/2 = \frac{-x - 1}{2}$

$2y + \frac{x}{x_1}^1 = -x - 1$

$2y = -x - 1$

∴ Equation of the normal =

$2y + x + 1 + 1 = 0$

∴ Equation of the tangent = $y - 2x - 3/2 = 0$