Course Lecturer : Dr. Otelami
Name : Rabiu Abdulazeez Olamilekan
Matric Number : 19/ENG07/019
Department : Petroleum Engineering
Date Submitted : Thursday, 2 April

1. Find the limit of the function $\frac{(x - \cos x)}{x}$ as $x \to 0$

~~$\lim x \to 0 = dy/dx$~~

$$\lim_{x\to 0}\left[\frac{1-\cos x}{x} \times \frac{1+\cos x}{1+\cos x}\right]$$

$$= \lim_{x\to 0}\left[\frac{1-\cos^2 x}{x(1+\cos x)}\right]$$

{ Recall $\sin^2 x + \cos^2 x = 1$
$\therefore 1 - \cos^2 x = \sin^2 x$

$$= \lim_{x\to 0}\left[\frac{\sin^2 x}{x(1+\cos x)}\right]$$

$$= \lim_{x\to 0}\left[\frac{\sin x}{x} \cdot \frac{\sin x}{1+\cos x}\right]$$

$$= \lim_{x\to 0}\left[\frac{\sin x}{x}\right] \cdot \lim_{x\to 0}\left[\frac{\sin x}{1+\cos x}\right]$$

$$= 1 \cdot 0$$

$$= 0$$

2. If $y = 3\tan 7x e 3x$ find dy/dx

$y = -3\tan 7x e^{3x}$

Let $u = -3\tan 7x$

$V = e^{3x}$

to find du/dx $\quad z = 7x$, $u = -3\tan z$

$$\frac{dz}{dx} = 7$$

$$\frac{du}{dz} = -3\sec^2 z$$

$$\frac{du}{dx} = \frac{du}{dz} \times \frac{dz}{dx}$$

$$= -3\sec^2 z \cdot 7$$
$$= -21\sec^2 z$$
$$= -21\sec^2 7x$$

To find $\frac{dv}{dx}$

$v = e^{3x}$

$z = 3x$

$dz/dx = 3$

$v = e^z$

$dv/dz = e^z$

$dv/dx = 3e^{3x}$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$= -3\tan 7x\,(3e^{3x}) + e^{3x}(-21\sec^2 7x)$$
$$= -9\tan 7x\, e^{3x} - 21\sec^2 7x\, e^{3x}$$
$$= -e^{3x}(9\tan 7x + 21\sec^2 7x)$$

~~If $y = 3\tan 7x e 3x$ find dy/dx~~

3. If $y = \cos 3x$ find dy/dx from first principle

$$y = \cos 3x$$
$$y + \Delta y = \cos 3(x + \Delta x)$$
$$\Delta y = \cos 3(x + \Delta x) - y$$
$$\Delta y = \cos 3(x + \Delta x) - \cos 3x$$
$$\Delta y = \cos 3x + 3\Delta x - \cos 3x$$
$$\frac{\Delta y}{\Delta x} = \frac{\cos 3x + 3\Delta x - \cos 3x}{\Delta x}$$

Recall, $\cos(A+B) = \cos A\cos B - \sin A \sin B$

$$\therefore \frac{\Delta y}{\Delta x} = \frac{\cos(3x)\cos(3\Delta x) - \sin(3x)\sin(3\Delta x) - \cos(3x)}{\Delta x}$$
$$\frac{\Delta y}{\Delta x} = \frac{\cos(3x)(\cos 3\Delta x - 1) - \sin(3x)\sin(3\Delta x)}{\Delta x}$$
$$\frac{\Delta y}{\Delta x} = \cos 3x\,\frac{(\cos 3\Delta x - 1)}{\Delta x} - \sin 3x\,\frac{\sin 3\Delta x}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \cos 3x \left(\frac{\cos 3\Delta x - 1}{\Delta x} \cdot \frac{\cos 3\Delta x + 1}{\cos 3\Delta x + 1} \right) - \sin 3x \cdot \frac{\sin 3\Delta x}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \cos 3x \left(\frac{\cos^2 3\Delta x - 1}{\Delta x(\cos 3\Delta x + 1)} \right) - \sin(3x) \cdot \frac{\sin \Delta x}{\Delta x}$$

Also $\sin^2\theta + \cos^2\theta = 1$

$\therefore \cos^2\theta - 1 = -\sin^2\theta$

$$\frac{\Delta y}{\Delta x} = -\cos 3x \left(\frac{\sin^2 \Delta x}{\Delta x(\cos 3\Delta x + 1)} \right) - \sin 3x \cdot \frac{\sin \Delta x}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \left(\frac{-\cos 3x \cdot \sin 3\Delta x - \sin 3x}{\cos 3\Delta x + 1} \right) \cdot \frac{\sin 3\Delta x}{\Delta x}$$

$$\lim_{x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{-\cos 3x \cdot \sin\theta}{\cos\theta + 1} - \sin 3x$$

$$\frac{dy}{dx} = \frac{-\cos 3x \times 0}{1+1} - \sin 3x$$

$$\frac{dy}{dx} = 0 - \sin 3x$$

$dy/dx = -\sin 3x$

4. Given $f(x) = 4x^2 + 2$ and $f(x) = 2x + 3$

$fog(x)$

$f(x) = 4x^2 + 2 \quad g(x) = 2x + 3$

$fog(x) = 4(2x+3)^2 + 2$

$= 4(4x^2 + 12x + 9) + 2$

$= 16x^2 + 48x + 36 + 2$

$\therefore fog\, x = 16x^2 + 48x + 38$

5. $f(x) = 4x^2 + 2$

$g(x) = 2x + 3$

To find $fog(x)$

$fog(x) = 4(2x+3)^2 + 2$

$= 4(2x+3)(2x+3) + 2$

$= 4(4x^2 + 6x + 6x + 9) + 2$

$= 16x^2 + 48x + 36 + 2$

$= 16x^2 + 48x + 38$ //

6. $x^2 + 2x + y^2 = 1020$

Gradient $= dy/dx$

differentiating implicity

$2x + 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} \neq 0$

$2x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 2y$

$(2x+2y)\,dy/dx = -(2x+2y)$

$dy/dx = \frac{-(2x+2y)}{(2x+2y)}$

$= -1$

7. $y = x^2 \cos x$

Finding the first derivataive

$u = x^2$

$v = \cos x$

$du/dx = 2x$

$dv/dx = -\sin x$

$dy/dx = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x^2(-\sin x) + \cos x(2x)$

$= -x^2 \sin x + 2x \cos x$

$= x(2\cos x - x\sin x)$