MAT 104
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1 Find the limit of the function $(x - \cos x)/x$ as $x \to 0$

Solution

using L'Hopital's rule we have $\frac{0}{0}$ undefined

$$\lim_{x \to 0}\left[\frac{x - \cos x}{x}\right] = \lim_{x \to 0}\left[\frac{1 - (-\sin x)}{1}\right]$$

$$= \frac{1 + \sin 0}{1} = \frac{1 + 0}{1} = \frac{1}{1} = 1$$

2 if $y = -3 \tan 7x\, e^{3x}$ find $dy/dx$

Solution

$u = -3\tan 7x$, $e^{3x} = v$

$$dy/dx = u\, dv/dx + v\, du/dx$$

$$dv/dx = e^{3x} \cdot 3e^{3x} \qquad du/dx = -21\sec^2 7x$$

$$= -3\tan 7x\,(3e^{3x}) + e^{3x}(-21\sec^2 7x)$$

3 If $y = \cos 3x$ find $dy/dx$ using first principle

$y = \cos 3x$

$y + \Delta y = \cos 3(x + \Delta x)$

$\Delta y = \cos 3(x + \Delta x) - y$

$\Delta y = \cos 3(x + \Delta x) - \cos 3x$

$\frac{dy}{dx}$ or $\cos 3x$

$$\frac{d \cos 3x}{dx} = \lim_{h \to 0} \frac{\cos(3(x+x)) - \cos(3x)}{h}$$

Also $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\cos(3x + 3h) - \cos(3x)}{h}$$

Hence, $\cos(3x + 3h) = \cos(3x)\cos(3h) - \sin(3x)\sin($

$$= \lim_{h \to 0} \frac{\cos(3x)\cos(3h) - \sin 3x \sin(3h) - \cos(3$$

$$= \lim_{h \to 0} \left( \cos(3x) \frac{\cos 3h - 1}{h} - \sin(3x) \frac{\sin 3h}{h} \right)$$

multiply by $\frac{\cos 3h + 1}{\cos 3h + 1}$

$$\lim_{h\to 0}\left(\frac{\cos(3x)\sin^2 3h + \sin(3x)\sin(3h)}{\cos(3h)\,h}\right)$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\lim_{h\to 0}\left(\frac{\cos(3x)\sin(3h) - \sin(3x)\left[\frac{\sin(3h)}{h}\right]}{\cos(3h)}\right)$$

$$\lim_{h\to 0}\left(\frac{-\cos(3x)\sin(3h) - \sin(3x)}{\cos(3h)}\right)\left(\lim_{h\to 0}\frac{\sin(3h)}{h}\right)$$

$$= \frac{\cos(3x)\sin(0) - \sin(3x)\cos(0)}{\cos(0)+1}\left(\lim_{h\to 0}\frac{\sin(3h)}{h}\right)$$

$$-\sin(3x)\left(\lim_{h\to 0}\frac{\sin 3h}{h}\right)$$

$$-\lim_{h\to}\frac{\sin 3h}{h} = 3$$

$$\frac{d(\cos 3x)}{dx} = -3\sin 3x$$

3. Given that $f(x) = 2x^3 - 7x$ and $g(x) = -3x$ find $(f-g)(5)$

$$(f-g) = 2x^3 - 7x - (-3)$$

$$= 2x^3 - 7x + 3x$$

$$= 2x^3 - 4x$$

$$(f-g)(5) = 2(5)^3 - 4(5)$$

$$= 250 - 20$$

$$= 230$$

5 find $f_{og}(x)$ if $f(x) = 4x^2 + 2$ and $g(x) = 2x+3$

Solution

$f_{og}(x) = 4(2x+3)^2 + 2$

$= 4[(2x+3)(2x+3)] + 2$

$= 4[4x^2 + 6x + 6x + 9] + 2$

$= 16x^2 + 24x + 24x + 36 + 2$

$= 16x^2 + 48x + 38$

6 find the gradient of $x^2 + 2xy + y^2 = 1020$

Solution

$2x\frac{dx}{dx} + \left[2y\frac{dx}{dx} + 2x\frac{dy}{dx}\right] + 2y\frac{dy}{dx} = 1020$

$2x + \left[2y + 2x\frac{dy}{dx}\right] + 2y\frac{dy}{dx} = 0$

$2x + 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = 1020$ 0

$2x + 2y + \frac{dy}{dx}[2x + 2y] = 1020$ 0

$\frac{dy}{dx} = \frac{-2x - 2y}{2x + 2y}$

7 find the first derivative of the function $y = x^2 \cos x$

$u = x^2$, $v = \cos x$

$u\frac{dv}{dx} + v\frac{du}{dx}$ $\frac{du}{dx} = 2x$ $\frac{dv}{dx} = -\sin x$

$= x^2(-\sin x) + \cos x(2x)$

$= -x^2 \sin x + 2x \cos x$