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Assignment

For the curves in problem 1 to 5, at the points given, find:

(a) the equation of the tangent and
(b) the equation of the normal

1. $y = 2x^2$ at the point $(1, 2)$
2. $y = 3x^2 - 2x$ at the point $(2, 8)$
3. $y = x^{3/2}$ at the point $(-1, -1/2)$
4. $y = 1 + x - x^2$ at the point $(-2, -5)$
5. $y = 1/x$ at the point $(3, 1/3)$

Solution

1. $y = 2x^2$ at the point $(1, 2)$

$$m = \frac{dy}{dx} = 4x$$

a) $\left.\frac{dy}{dx}\right|_{x=1}$

$m = 4,\ x = 1,\ y = 2$

$y - y_1 = m(x - x_1)$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y - 2 - 4x + 4 = 0$

$y - 4x + 2 = 0$ [Equation of the tangent]

b) The equation of the normal

$m_1 m_2 = -1$

$$m_2 = \frac{-1}{m_1} = \frac{-1}{4}$$

$y - y_1 = m_2(x - x_1)$

$y - 2 = -1/4\,(x - 1)$

$4y - 8 = -x + 1$

$4y - 8 + x - 1 = 0$

$4y + x - 9 = 0$ [Equation of the normal]

2) $y = 3x^2 - 2x$ at the point $(2, 8)$

$m = \frac{dy}{dx} = 6x - 2$

$\frac{dy}{dx}\Big|_{x=2} \quad 6(2) - 2 = 12 - 2 = 10$

b)

$m = 10, \quad x = 2, \quad y = 8$

a) $y - y_1 = m(x - x_1)$

$y - 8 = 10(x - 2)$

$y - 8 = 10x - 20$

$y - 8 - 10x + 20 = 0$

$y - 10x + 12 = 0$ [Equation of the tangent]

b) Equation of the normal

$m_1 m_2 = -1$

$m_2 = \frac{-1}{m_1} = \frac{-1}{10}$

$y - y_1 = m(x - x_1)$

$y - 8 = \frac{-1}{10}(x - 2)$

4)

$10y - 80 = -x + 2$

$10y - 80 + x - 2 = 0$

$10y + x - 82 = 0$ [Equation of the normal]

3) $y = x^3/2$ at the point $(-1, -1/2)$.

$m = \frac{dy}{dx} = \frac{3x^2}{2}$

a)

$\frac{dy}{dx}\Big|_{x=-1} = \frac{3}{2}(-1)^2 = \frac{3}{2}$

$m = \frac{3}{2}, \quad x = -1, \quad y = \frac{-1}{2}$

a) $y - y_1 = m(x - x_1)$

$y - (-1/2) = \frac{3}{2}(x - (-1))$

b)

$y + \frac{1}{2} = \frac{3}{2}(x + 1)$

$2(y + 1/2) = 3(x + 1)$

$2y + 1 = 3x + 3$

$2y + 1 - 3x - 3 = 0$

$2y - 3x - 2 = 0$ (Equation of the tangent)

b) $m_1 m_2 = -1$

$m_2 = \frac{-1}{m_1} = -1 \div \frac{3}{2} = \frac{-2}{3}$

$y - y_1 = m_2(x - x_1)$

$y - (-1/2) = -2/3 (x - (-1))$

$y + 1/2 = -2/3 (x + 1)$

$3y + \frac{3}{2} = -2x - 1$

$3y + \frac{3}{2} + 2x + 2 = 0$

$3y + 2x + \frac{7}{2} = 0$ [Equation of the normal]

4) $y = 1 + x - x^2$ at the point $(-2, -5)$

$m = \frac{dy}{dx} = 1 - 2x$

$m = \frac{dy}{dx}\Big|_{x=-2} = 1 - 2(-2) = 1 + 4 = 5$

$m = 5, \quad x = -2, \quad y = -5$

a) $y - y_1 = m(x - x_1)$

$y - (-5) = 5(x - (-2))$

$y + 5 = 5(x + 2)$

$y + 5 = 5x + 10$

$y + 5 = 5x + 10$

$y + 5 - 5x - 10 = 0$

$y - 5x - 5 = 0$ [Equation of the tangent].

b) $m_1 m_2 = -1$

$m_2 = \frac{-1}{m_1} = \frac{-1}{5}$

$y - y_1 = m_2 (x - x_1$

$y + 5 = \frac{-1}{5} (x + 2)$

$5y + 25 = -x - 2$

$5y + 25 + x + 2 = 0$

$5y + x + 27 = 0$ [Equation of the normal]

5) $y = 1/x$ at the point $[3, 1/3]$

$\frac{dy}{dx} = x^{-1} = -1 \times x^{-1-1} = -1 x^{-2} = -\bar{x}^2 \therefore m = -x^{-2}$

$\frac{dy}{dx}\Big|_{x=3} = -x^{-2} = (-3)^2 = \frac{1}{(-3)^2} = \frac{1}{9}$

$m = \frac{1}{9}, x = 3, y = \frac{1}{3}$

a) $y - y_1 = m(x - x_1)$

$y - \frac{1}{3} = \frac{1}{9} (x - 3)$

$9y - 3 = x - 3$

$9y - 3 = x - 3$

$9y - 3 - x + 3 = 0$

$9y - x = 0$ [Equation of the tangent]

b) $m_2 = \frac{-1}{m_1} = -1 \div \frac{1}{9} = -9$

$y - y_1 = m_2 (x - x_1)$

$y - \frac{1}{3} = -9 (x - 3)$

$y - \frac{1}{3} = -9 (x - 3)$

$y - \frac{1}{3} = -9x + 27$

$y - \frac{1}{3} + 9x - 27 = 0$

$y + 9x - \frac{82}{3} = 0$

$3y + 27x - 82 = 0$ [Equation of the normal]