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ASSIGNMENT TITLE: EQUATION OF TANGENT & NORMAL
COURSE TITLE: General mathematics III
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Question

For the curves in problem 1 to 5, at the points given, find (a) the equation of the tangent, and (b) the equation of the normal.

1) $y = 2x^2$ at the point $(1, 2)$
2) $y = 3x^2 - 2x$ at the point $(2, 8)$
3) $y = x^{3/2}$ at the point $(-1, -1/2)$
4) $y = 1 + x - x^2$ at the point $(-2, -5)$
5) $y = 1/x$ at the point $(3, 1/3)$

Solution

1) $y = 2x^2$ at point $(1, 2)$

$\frac{dy}{dx} = 4x$

$m = \left.\frac{dy}{dx}\right|_{x=1} = 4(1) = 4$

Equation of tangent

$y - y_1 = m(x - x_1)$

$y - (2) = 4(x - 1)$

$y - 2 = 4x - 4$

$y - 4x - 2 + 4 = 0$

$y - 4x + 2 = 0$

Equation of normal, =

$m_2 = \frac{1}{m_1} = \frac{-1}{4} = \frac{-1}{4}$

$y - y_1 = m_2(x - x_1)$

$y - (2) = \frac{-1}{4}(x - 1)$

$y - 2 = \frac{-1}{4}(x - 1)$

$4y - 8 = -x + 1$

$4y + x - 8 - 1 = 0$

$4y + x - 9 = 0$

2.) $y = 3x^2 - 2x$ at the point $(2, 8)$

$\frac{dy}{dx} = 6x - 2$

$m = \left.\frac{dy}{dx}\right|_{x=2} = 6(2) - 2 = 12 - 2 = 10$

Equation of tangent,

$y - y_1 = m(x - x_1)$

$y - 8 = 10(x - 2)$

$y - 8 = 10x - 20$

$y - 10x - 8 + 20 = 0$

$y - 10x + 12 = 0$

Equation of normal,

$m_2 = \frac{-1}{m_1} = \frac{-1}{10}$

$y - y_1 = m(x - x_1)$

$y - 8 = \frac{-1}{10}(x - 2)$

$10(y - 8) = -1(x - 2)$

$10y - 80 = -x + 2$

$10y + x - 80 - 2 = 0$

$10y + x - 82 = 0$

3.) $y = \frac{x^3}{2}$ at the point $(-1, -\frac{1}{2})$

$\frac{dy}{dx} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2} = \frac{2(3x^2) - x^3(0)}{2^2}$

$\frac{dy}{dx} = \frac{6x^2}{4} = \frac{3x^2}{2}$

$m = \frac{dy}{dx}\bigg|_{x=-1} = \frac{3(-1)^2}{2} = \frac{3}{2}$

Equation of the tangent =

$$y - y_1 = m(x - x_1)$$

$$y - (-\tfrac{1}{2}) = \tfrac{3}{2}(x - (-1))$$

$$\frac{y}{1} + \frac{1}{2} = \frac{3}{2}(x + 1)$$

$$\frac{2y + 1}{2} = \frac{3}{2}(x + 1)$$

$$2(2y + 1) = 2(3x + 3)$$

$$4y + 2 = 6x + 6$$

$$4y - 6x + 2 - 6 = 0$$

$$4y - 6x - 4 = 0$$

$$2y - 3x - 2 = 0$$

Equation of the normal =

$$m_2 = \frac{-1}{m_1} = -1 \div \frac{3}{2} = -1 \times \frac{2}{3} = -\frac{2}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - (-\tfrac{1}{2}) = \frac{-2}{3}(x - (-1))$$

$$\frac{y}{1} + \frac{1}{2} = \frac{-2}{3}(x + 1)$$

$$\frac{2y + 1}{2} = \frac{-2x - 2}{3}$$

$$3(2y + 1) = 2(-2x - 2)$$

$6y + 3 = -4x - 4$

$6y + 4x + 3 + 4 = 0$

$6y + 4x + 7 = 0$

4.) $y = 1 + x - x^2$ at the point $(-2, -5)$

$\frac{dy}{dx} = 1 - 2x$

$m = \frac{dy}{dx}\Big|_{x=-2} = 1 - 2(-2) = 5$

Equation of the tangent =

$y - y_1 = m(x - x_1)$

$y - (-5) = 5(x - (-2))$

$y + 5 = 5(x + 2)$

$y + 5 = 5x + 10$

$y - 5x + 5 - 10 = 0$

$y - 5x - 5 = 0$

Equation of the normal

$m_2 = \frac{-1}{m_1} = \frac{-1}{5}$

$y - y_1 = m_2(x - x_1)$

$y - (-5) = \frac{-1}{5}(x - (-2))$

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$y + 5 = \frac{-1}{5}(x + 2)$

$5(y+5) = -1(x+2)$

$5y + 25 = -x - 2$

$5y + x + 25 + 2 = 0$

$5y + x + 27 = 0$

5.) $y = \frac{1}{x}$ at the point $(3, \frac{1}{3})$

$y = x^{-1}$

$\frac{dy}{dy} = -1x^{-2} = -x^{-2} = \frac{-1}{x^2}$

$m = \frac{dy}{dx}\Big|_{x=3} = \frac{-1}{x^2} = \frac{-1}{(3)^2} = \frac{-1}{9}$

Equation of the tangent, =

$y - y_1 = m(x - x_1)$

$y - (\frac{1}{3}) = \frac{-1}{9}(x-3)$

$\frac{y}{1} - \frac{1}{3} = \frac{-1}{9}(x-3)$

$\frac{3y-1}{3} = \frac{-x+3}{9}$

$9(3y-1) = 3(-x+3)$

$27y - 9 = -3x + 9$

$27y + 3x - 9 - 9 = 0$

$27y + 3x - 18 = 0$

$9y + x - 6 = 0$

Equation of the normal, =

$m_2 = \frac{-1}{m_1} = -1 \div \frac{-1}{9} = -1 \times \frac{9}{-1} = 9$

$y - y_1 = m(x - x_1)$

$y - (1/3) = 9(x - 3)$

$\frac{y}{1} - \frac{1}{3} = 9x - 27$

$\frac{3y - 1}{3} = 9x - 27$

$3y - 1 = 3(9x - 27)$

$3y - 1 = 27x - 81$

$3y - 27x - 1 + 81 = 0$

$3y - 27x + 80 = 0$