AFINIKI JOHN

Computer Engineering

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MAT 104

Assignment

Differentiate

1 $y = \frac{[(x+1)^2 (x-2)]^{1/2}}{[(2x-1)(x+3)]^{4/3}}$

$\ln y = [2\ln(x+1) + \tfrac{1}{2}\ln(x-2)] - [\ln(2x-1) + \tfrac{3}{2}\ln(x+3)]$

$\frac{1}{y} \cdot \frac{dy}{dx} = \left(2 \times \frac{1}{x+1} \times 1 + \frac{1}{2} \times \frac{1}{x-2} \times 1\right) - \left(\frac{1}{2x-1} \times \frac{2}{1} + \frac{3}{2} \times \frac{1}{x+3} \cdot 1\right)$

$\frac{1}{y} \cdot \frac{dy}{dx} = \left(\frac{2}{x+1} + \frac{1}{2(x-2)}\right) - \left(\frac{2}{2x-1} + \frac{3}{2(x+3)}\right)$

$\frac{dy}{dx} = y\left[\frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{3}{2(x+3)}\right]$

$\therefore \frac{dy}{dx} = \frac{[(x+1)^2(x-2)]^{1/2}}{[(2x-1)(x+3)]^{3/2}}\left[\frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{3}{2x+3}\right]$

2 $y = \frac{[3e^x \sin 2k]}{k^{5/2}}$

$\ln y = (\ln 3e^x) + \ln(\sin 2k) - \tfrac{5}{2}\ln k$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{3e^x} \times 3e^x + \frac{1}{\sin 2k} \times 2\cos 2k - \frac{5}{2} \times \frac{1}{k}$

$\frac{1}{y} \cdot \frac{dy}{dx} = 1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2k}$

$\frac{dy}{dx} = y\left[1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2k}\right]$

$\frac{dy}{dx} = \frac{[3e^x \sin 2k]}{k^{5/2}}\left[1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2k}\right]$

Integration with respect to the Variable.

1 $4\sec^2(3m+1)$

$\int 4\sec^2(3m+1)\,dm$

$4\int \sec^2(3m+1)\,dm$

$u = 3m+1$

$\therefore \frac{du}{dm} = 3$

$dm = \frac{du}{3}$

$\frac{4}{3}\int \sec^2 u\,du$

$\frac{4}{3}(\tan u) + c$

$= \frac{4}{3}\tan(3m+1) + c$

$\int 4\sec^2(3m+1)\,dm = \frac{4}{3}\tan(3m+1) + c$

2 $2t(3t^2-1)^{1/2}$

$\int 2t(3t^2-1)^{1/2}\,dt$

$2\int t(3t^2-1)^{1/2}\,dt$

$u = (3t^2-1)^{1/2}$

$u^2 = 3t^2-1$

$u^2+1 = 3t^2$

$t^2 = \frac{u^2+1}{3}$

$t = \left(\frac{u^2+1}{3}\right)^{1/2}$

$\frac{dt}{du} = \frac{1}{2}\left(\frac{u^2+1}{3}\right)^{-1/2} \times \frac{2u}{3}$

$\frac{dt}{du} = \frac{u}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$dt = \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$$2\int\left(\frac{u^2+1}{3}\right)^{1/2} \times u \times \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$$

$$\frac{2}{3}\int\left(\frac{u^2+1}{3}\right)^{1/2-1/2} \times u^2\,du$$

$$\frac{2}{3}\int u^2\,du$$

$$\frac{2}{3}\left[\frac{u^3}{3}\right]+C$$

$$=\frac{2(3t^2-1)^{3/2}}{3}+C$$

$$\int 2t(3t^2-1)^{1/2}\,dt = \frac{2(3t^2-1)^{3/2}}{3}+C$$

3 $\dfrac{2x}{(4x^2-1)^{1/2}}$

$$2\int\frac{u}{\sqrt{4x^2-1}}\,dx$$

$$u=\sqrt{4x^2-1}$$

$$u^2+1=4x^2$$

$$x=\left(\frac{u^2+1}{4}\right)^{1/2}$$

$$\frac{dx}{du}=\frac{1}{2}\left(\frac{u^2+1}{4}\right)^{-1/2}\times\frac{u}{2}$$

$$du=\frac{u\,du}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$$

$$2\int\left(\frac{u^2+1}{4}\right)^{1/2}\times\frac{1}{u}\times\frac{u\,du}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$$

$$=\frac{2}{4}\int\left(\frac{u+1}{4}\right)^{1/2-1/2}\times\frac{1}{u}\,u\,du$$

$$\frac{1}{2}\int du$$

$$\frac{1}{2}(u)+C$$

$$\int\frac{2x}{(4x^2-1)^{1/2}}\,dx=\frac{\sqrt{4x^2-1}}{2}+C$$