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19/ENG05/023
MECHATRONIC ENGINEERING

MATH 104
Dr OYelomi's Group

1 Find the Limit of the function
$(x-\cos x)/x$ } $x \to 0$

$\lim x \to 0 = dy/dx$

$y = (x-\cos x)/x$

let $u = x - \cos x$

$v = x$

$du/dx = 1-(-\sin x) = 1+\sin x$

$dv/dx = 1$

$\frac{dy}{dx} = \frac{v\,du/dx - u\,dv/dx}{v^2}$

$= x(1+\sin x) - (x-\cos x)$
x^2

$= \frac{x + x\sin x - x + \cos x}{x^2}$

$\frac{x\sin x + \cos x}{x^2}$

2 If $y = -3\tan 7x e^{3x}$ find dy/dx

$y = -3\tan 7x\, e^{3x}$

$u = -3\tan 7x$

$v = e^{3x}$

to find dv/dx

$z = 7x$

$u = -3\tan z$

$\frac{dz}{dx} = 7$

$\frac{du}{dz} = -3\sec^2 z$

$\frac{du}{dx} = \frac{du}{dz} \times \frac{dz}{dx}$

$-3\sec^2 z \cdot 7$

$-21\sec^2 z$

$-21\sec^2 7x$

To find $\frac{dv}{dx}$

$V = e^{3x}$

$z = 3x$

$dz/dx = 3$

$V = e^z$

$dv/dz = e^z$

$dv/dx = 3e^{3x}$

$\frac{dy}{dx} = U\frac{dv}{dx} + V\frac{dy}{dx}$

$= -\tan 7x(3e^{3x}) + e^{3x}(-21\sec^2 7x)$

$-9\tan 7x\, e^{3x} - 21\sec^2 7x\, e^{3x}$

$-e^{3x}(9\tan 7x + 21\sec^2 7x)$

3 If $y = \cos 3x$, Find dy/dx from the first principle

$y = \cos 3x$

Finding dy/dx from the first Principle

$y = \cos 3x$

$f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$\frac{dy}{dx}\cos(3x) = \lim_{\Delta x \to 0} \frac{\cos 3(x+\Delta x) - \cos 3x}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{\cos 3(3x + 3\Delta x) - \cos 3x}{\Delta x}$

$\cos(3x + 3\Delta x = \cos(3x)\cos(3\Delta x) - \sin(3x)\sin(3\Delta x)$

$\lim_{\Delta x \to 0} = \frac{\cos(3x)\cos(3\Delta x) - \sin(3x)\sin(3\Delta x) - \cos(3x)}{\Delta x}$

$$\lim_{\Delta x \to 0} \frac{\cos(3x)\cos(3\Delta x) - \cos(3x) - \sin(3x)\sin(3\Delta x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \left(\frac{\cos(3x)(\cos(3\Delta x) - 1)}{\Delta x} - \sin(3x)\frac{(\sin(3\Delta x)}{\Delta x} \right)$$

multiplying $\frac{\cos(3\Delta x - 1)}{\Delta x}$ by $\frac{\cos(3\Delta x + 1)}{\cos(3\Delta x + 1)}$

$$\lim_{\Delta x \to 0} \left(\frac{\cos(3x)(\cos^2(3\Delta x) - 1)}{\cos(3\Delta x + 1)\Delta x} - \sin(3x)\frac{\sin(3\Delta x)}{\Delta x} \right)$$

recall $\cos^2 x - 1 = \sin^2 x$

$$\lim_{\Delta x \to 0} \left(\frac{-\cos(3x)(\sin^2(3\Delta x)}{\cos(3\Delta x + 1)\Delta x} - \sin(3x)\frac{(\sin(3\Delta x)}{\Delta x} \right)$$

$$\lim_{\Delta x \to 0} \left[\left(\frac{-\cos(3x)(\sin(3\Delta x)}{\cos(3\Delta x + 1)} - \sin(3x) \right) \frac{\sin(3\Delta x)}{\Delta x} \right.$$

As $\Delta x \to 0$

$$-\sin(3x) \lim_{\Delta x \to 0} \frac{\sin(3\Delta x)}{\Delta x}$$

$$dy/dx = -3\sin(3x)$$

4. Given that $f(x) = 2x^3 - 7x$ and $g(x) = -3x$ find $(f-g)(5)$

$f(x) = 2x^3 - 7x$

$g(x) = -3x$

Find $(f-g)(5)$

$f(x) - g(x)$

$2x^3 - 7x - (-3x)$

$2x^3 - 4x$

$(f-g) = 2x^3 - 4x$

$(f-g)(5) = 2(5)^3 - 4(5)$

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5 Find fog(x) If $f(x) = 4x^2 + 2$ and $g(x) = 2x + 3$

$f(x) = 4x^2 + 2$

$g(x) = 2x + 3$

fog(x)

$fog(x) = 4(2x+3)^2 + 2$

$4(2x+3)(2x+3) + 2$

$4(4x^2 + 6x + 6x + 9) + 2$

$4(4x^2 + 12x + 9) + 2$

$16x^2 + 48x + 36 + 2$

$16x^2 + 48x + 38$

6 Find the gradient of $x^2 + 2xy + y^2 = 1020$

$x^2 + 2xy + y^2 = 1020$

gradient $= \frac{dy}{dx}$

differentiating implicitly

$2x + 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$2x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 2y$

$(2x + 2y)\,dy/dx = -(2x + 2y)$

$dy/dx = \frac{-(2x+2y)}{2x+2y}$

$= -1$

7 Find the first derivative of the fantion $y = x^2 \cos x$

$y = x^2 \cos x$

finfind the ~~first~~ first derivative

$u = x^2$

$v = \cos x$

7 $du/dx = 2x$

$dv/dx = -\sin x$

$u\frac{dv}{dx} + V\frac{dy}{dx} = \frac{dy}{dx}$

~~$x^2(-\sin x)$~~ $x^2(-\sin x) + \cos x(2x)$

$-x^2 \sin x + 2x \cos x$

$x(2\cos x - x \sin x)$