NAME OF COURSE LECTURER: DR OYELAMI COURSE CODE: MAT104
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MATRIC NO: 19/ENG08/002, COURSE TITLE: GENERAL MATHEMATICS III
ASSIGNMENT TITLE: ASSIGNMENT FOR DR OYELAMI'S GROUP

1 Find the limit of the function $\{(x-\cos x)/x\}$ as $x \to 0$

Solution

Given $\frac{x-\cos x}{x}$ by direct substitution, we have

$$\lim_{x \to 0} \frac{x-\cos x}{x} = \frac{0-\cos 0}{0} = \frac{0-1}{0} = \frac{-1}{0} = \text{undefined}$$

Using L hopital rule for differensiating

$$\lim_{x \to 0} \left(\frac{x-\cos x}{x}\right) = \frac{1-(-\sin x)}{1} = \frac{1+\sin x}{1} = 1+\sin x$$

$$\therefore \lim_{x \to 0} (1+\sin x) = 1+\sin 0 = 1+0 = 1 \;//$$

2 If $y = -3\tan 7x\, e^{3x}$; find dy/dx

Solution

Given $y = -3\tan 7x e^{3x}$

let $u = -3\tan 7x$ and $v = e^{3x}$

using quotient rule

$$\frac{du}{dx} = -21\sec^2 7x, \quad \frac{dv}{dx} = -3e^{3x}$$

$$\therefore \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = -3\tan 7x - 3e^{3x} + e^{3x} - 21\sec^2 7x$$

$$= 3e^{3x}(-3\tan 7x) + e^{3x}(-21\sec^2 7x)$$

3. If $y = \cos 3x$, find dy/dx from the first principle

Solution

$$y + \Delta y = \cos 3(x + \Delta x)$$

$$\Delta y = \cos 3(x + \Delta x) - \cos 3x \quad \text{— (i)}$$

Recall $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ — (ii)

Comparing equation (i) & (ii)

$$A = 3(x + dx), \quad B = 3x$$

$$\frac{A+B}{2} = \frac{3(x+\Delta x)+3x}{2} = \frac{3x + 3\Delta x + 3x}{2} = \frac{6x + 3\Delta x}{2}$$

$$\frac{A+B}{2} = \frac{\cancel{6x}}{\cancel{2}} + \frac{3\Delta x}{2} = 3x + \frac{3\Delta x}{2}$$

Also

$$\frac{A-B}{2} = \frac{3(x+\Delta x)-3x}{2} = \frac{3x+3\Delta x-3x}{2} = \frac{3\Delta x}{2}$$

Hence $\Delta y = -2\sin\left(\frac{3x + 3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)$

$$\frac{\Delta y}{\Delta x} = \frac{-2\sin\left(3x + \frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)}{\Delta x}$$

Multiply both numerator and denominator by $1/2$

$$\frac{\Delta y}{\Delta x} = \frac{-2\sin\left(3x + \frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right) \times \frac{1}{2}}{\Delta x \times \frac{1}{2}}$$

$$\frac{\Delta y}{\Delta x} = -\sin\left(3x + \frac{3\Delta x}{2}\right) \frac{\sin\frac{3\Delta x}{2}}{\frac{\Delta x}{2}}$$

taking limit as $\Delta x \to 0$

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = -\sin 3x \qquad \lim_{\Delta x \to 0}\frac{\sin\left(\frac{3\Delta x}{2}\right)}{\frac{\Delta x}{2}} = 1$$

Hence

$$\frac{dy}{dx} = -\sin 3x$$

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4. Given that $f(x) = 2x^3 - 7x$ and $g(x) = -3x$ find $(f-g)5$

Solution

$f(5) = 2(5)^3 - 7(5)$
$= 2(125) - 35$
$= 250 - 35$
$= 215$

$g(5) = -3(5)$
$= -15$

$(f-g)(5) = (f)5 - g(5)$
$= 215 - (-15)$
$= 215 + 15$
$= 230$ //

5. Find the $f \circ g(x)$ if $f(x) = 4x^2 + 2$ and $g(x) = 2x + 3$

Solution

$f \circ g(x) = 4(2x+3)^2 + 2$
$= 4(2x+3)(2x+3) + 2$
$= 4\{4x^2 + 6x + 6x + 9\} + 2$
$= 4\{4x^2 + 12x + 9\} + 2$
$= 16x^2 + 48x + 36 + 2$
$= 16x^2 + 48x + 38 + 2$

$f \circ g(x) = 16x^2 + 48x + 38$ //

6. Find the gradient of $x^2 + 2xy + y^2 = 1020$

Solution

$x^2 + 2xy + y^2 = 1020$

Using Implicit differenciation since gradient $m = \frac{dy}{dx}$

$\therefore 2x\frac{dx}{dx} + 2\left(x\frac{dy}{dx} + y\frac{dx}{dx}\right) + 2y\frac{dy}{dx} = 0$

$2x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 0$

$2x + \frac{dy}{dx}(2x + 2y) + 2y = 0$

$\frac{dy}{dx}(2x + 2y) = -2x - 2y$

divide through by $(2x + 2y)$ to get dy/dx

$\frac{dy}{dx} = \frac{-2x - 2y}{2x + 2y}$

$= \frac{-2(x+y)}{2(x+y)} = \frac{-x-y}{x+y}$ //

The gradient $m = \frac{-x-y}{x+y}$ //

1) Find the first ~~derivative~~ derivative of the function $y = x^2\cos x$

Solution

$y = x^2\cos x$

$\frac{dy}{dx} = -2x\sin x$

The first derivative $= -2x\sin x$