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MAT 104 Assignment.

* differentiate the following.

① $y = \left[\dfrac{(x+1)^2 (x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}\right]$

Soln [using Logarithm of Diff]

$\ln y = \left[\ln\left[(x+1)^2\right] + \ln\left[(x-2)^{1/2}\right]\right] - \left[\ln[2x-1] + \ln(x-3)^{4/3}\right]$

$\frac{1}{y}\frac{dy}{dx} = \left[\frac{1}{(x+1)^2}\cdot 2(x+1) + \frac{1}{(x-2)^{1/2}}\cdot \frac{1}{2}(x-2)^{-1/2} - \frac{1}{(2x-1)}\cdot 2 - \frac{1}{(x-3)^{4/3}}\cdot \frac{4}{3}(x-3)^{1/3}\right]$

$= \frac{1}{y}\frac{dy}{dx} = \left[\frac{2}{x+1} + \frac{\frac{1}{\sqrt{x-2}}}{2\sqrt{x-2}} - \frac{2}{2x-1} - \frac{4(x-3)^{1/3-4/3}}{3}\right]$

$\therefore \frac{dy}{dx} = y\left[\frac{2}{x+1} + \frac{1}{2(x-2)}\right] - \left[\frac{2}{2x-1} - \frac{4}{3(x-3)}\right]$

$\frac{dy}{dx} = \frac{(x+1)^2(x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}\left[\frac{2}{(x+1)} + \frac{1}{2x-4} - \frac{2}{(2x-1)} - \frac{4}{3x-9}\right]$ //

②

$y = \frac{\left[3e^k \sin 2k\right]}{k^{5/2}}$

Soln [using logarithm of Diff]

$\ln y = \ln 3e^k + \ln \sin 2k - \ln k^{5/2}$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{3e^k}\cdot 3e^k + \frac{1}{\sin 2k}\cdot 2\cos 2k - \frac{1}{k^{5/2}}\cdot \frac{5}{2}k^{3/2}$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{3e^k}{3e^k} + \frac{2\cos 2k}{\sin 2k} - \frac{5\, k^{3/2}}{2\, k^{5/2}}$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \left[1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2}k^{3/2 - 5/2}\right]$$

$$\therefore \frac{dy}{dx} = y\left[1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2}k^{-1}\right]$$

$$\frac{dy}{dx} = \frac{3e^{k}\sin 2k}{k^{5/2}}\left[1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2k}\right]_{//}$$

* Integration [Integrate the following with respect to the variable].

(1) $4\sec^2(3m+1)$

Solu

$$4\int \sec^2(3m+1)\,dm$$

$\therefore (3m+1) = u \quad \therefore \frac{du}{dm} = \frac{3}{1} = 3dm \quad \therefore \frac{du}{3} = \frac{\cancel{3}dm}{\cancel{3}}$

$\therefore dm = \frac{du}{3}$

$$\therefore 4\int \sec^2(u)\frac{du}{3} = \frac{4}{3}\tan u + C$$

$$= \frac{4}{3}\tan(3m+1) + C$$

(2)

* $2t(3t^2-1)^{1/2}$

Solu

$$\int 2t(3t^2-1)^{1/2}\,dt$$

$\therefore$ let $u = \sqrt{3t^2-1} \quad \therefore u^2 = 3t^2-1 \quad \therefore 3t^2 = u^2+1$

$\therefore t^2 = \frac{u^2+1}{3} \quad \therefore t = \sqrt{\frac{u^2+1}{3}}$

$$\frac{dt}{du} = \frac{1}{2}\left(\frac{u^2+1}{3}\right)^{-1/2} \cdot \frac{2u}{3}$$

$$\frac{dt}{du} = \frac{u}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$$

$$dt = \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$$

$$\int 2\left(\frac{u^2+1}{3}\right)^{1/2} . u . \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$$

$$= \frac{2}{3}\int u^2 \left(\frac{u^2+1}{3}\right)^{1/2 - 1/2} du$$

$$= \frac{2}{3}\int u^2\, du$$

$$= \frac{2}{3}\left[\frac{u^3}{3}\right] + C$$

$$= \frac{2u^3}{9} + C$$

$$= \frac{2(3t^2-1)^{3/2}}{9} + C$$

3

$$\int \frac{2x}{[4x^2-1]^{1/2}}\, dx$$

Solu

$u = \sqrt{4x^2-1}$ $\therefore u^2 = 4x^2 - 1 = 4x^2 = u+1$

$$= \frac{2u^3}{9} + C$$

$$= \frac{2(3t^2-1)^{3/2}}{9} + C$$

③

$$\int \frac{2x}{[4x^2-1]^{1/2}}\,dx$$

Soln

$\cancel{u = \sqrt{4x^2-1} \quad \therefore u^2 = 4x^2 - 1 = 4x^2 = u+1 \quad \therefore x^2 = \frac{u+1}{4}}$

$\cancel{\therefore x = \sqrt{\frac{u+1}{4}}} \quad \therefore$

Soln

Let $u = 4x^2 - 1 \quad \therefore \frac{du}{dx} = 8x$

$dx = \frac{du}{8x} \quad = \int 2x \cdot (u)^{-1/2} \cdot \frac{du}{8x}$

$= \frac{1}{4}\int u^{-1/2}\,du = \frac{1}{4} \times \frac{u^{-1/2+1}}{-1/2+1} = \frac{1}{4} \times \frac{u^{1/2}}{1/2} = \frac{1}{4} \times 2u^{1/2}$

$\frac{1}{2}u^{1/2} = \frac{1}{2}(4x^2-1)^{1/2} \quad \therefore \boxed{\frac{1}{2}(4x^2-1)^{1/2}}$