

Differentiate the following

$$1) y = \frac{[(x+1)^2(x-2)^{1/2}]}{[(2x-1)(x-3)^{4/3}]}$$

$$\ln y = \ln[(x+1)^2] + \ln[(x-2)^{1/2}] - \ln(2x-1) - \ln[(x-3)^{4/3}]$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{(x+1)^2} \cdot 2x+2 + \frac{1}{(x-2)^{1/2}} \cdot \frac{1}{2}(x-2)^{-1/2} - \frac{1}{(2x-1)} - \frac{1}{(x-3)^{4/3}} \cdot \frac{4}{3}(x-3)^{1/3}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{2x+2}{(x+1)^2} + \frac{(x-2)^{-1/2}}{(x-2)^{1/2}} - \frac{1}{(2x-1)} - \frac{4/3(x-3)^{1/3}}{(x-3)^{4/3}} = \frac{2}{(2x-1)}$$

~~$\frac{1}{y} \frac{dy}{dx} = \frac{2x}{(x+1)^2} + \frac{1}{2(x-2)^{1/2}} - \frac{1}{2x-1} - \frac{4}{3(x-3)^{1/3}}$~~

$$\frac{1}{y} \frac{dy}{dx} = \frac{2}{(x+1)} + \frac{1}{2} - \frac{2}{(2x-1)} - \frac{4/3(x-3)^{1/3}}{(x-3)^{4/3}}$$

$$\frac{dy}{dx} = y \left[ \frac{2}{(x+1)} + \frac{1}{2} - \frac{2}{(2x-1)} - \frac{4/3(x-3)^{1/3}}{(x-3)^{4/3}} \right]$$

$$\frac{dy}{dx} = \frac{(x+1)^2(x-2)^{1/2}}{(2x-1)(x-3)^{4/3}} \left[ \frac{2}{(x+1)} + \frac{1}{2} - \frac{2}{(2x-1)} - \frac{4/3(x-3)^{1/3}}{(x-3)^{4/3}} \right]$$

$$2) y = \frac{[3e^k \sin 2k]}{k^{5/2}}$$

$$\ln y = \ln(3e^k) + \ln(\sin 2k) - \ln(k^{5/2})$$

$$\frac{1}{y} \frac{dy}{dk} = \frac{1}{3e^k} \cdot 3e^k + \frac{1}{\sin 2k} \cdot 2 \cos 2k - \frac{1}{k^{5/2}} \cdot \frac{5}{2} k^{3/2}$$

$$\frac{dy}{dk} = 1 + \frac{2 \cos 2k}{\sin 2k} - \frac{5k^{3/2}}{2k^{5/2}}$$

$$\frac{1}{y} \frac{dy}{dx} = 1 + 2 \tan 2k - \frac{5k^{3/2}}{2k^{5/2}}$$

$$\frac{dy}{dk} = y \left[ 1 + 2 \tan 2k - \frac{5k^{3/2}}{2k^{5/2}} \right]$$

$$k \frac{dy}{dk} = \frac{3e^x \sin 2k}{k^{5/2}} \left[ 1 + 2 \tan 2k - \frac{5k^{3/2}}{2k^{5/2}} \right]$$

Integration

1)  $\int 4 \sec^2(3m+1)$

let  $u = 3m+1$

$$\frac{du}{dm} = 3 \quad dx = \frac{du}{3}$$

$$= \int 4 \sec^2 u \cdot \frac{du}{3} = \frac{1}{3} \int 4 \sec^2 u \, du$$

$$= \frac{1}{3} \cdot [4 \tan u + c] = \frac{4 \tan u}{3} + c$$

$$\Rightarrow \frac{4 \tan(3m+1)}{3} + c //$$

2)  $\int 2t(3t^2-1)^{1/2} dt$

let  $u = 3t^2 - 1$

$$du = 6t \quad dt = \frac{du}{6t}$$

$$\Rightarrow \int 2t u^{1/2} \frac{du}{6t} = \int \frac{u^{1/2} du}{3}$$

$$= \frac{1}{3} \int u^{1/2} du \Rightarrow \frac{1}{3} \left[ \frac{u^{3/2}}{3/2} + c \right]$$

$$= \frac{2u^{3/2}}{9} + c$$

$$\Rightarrow \frac{2(3t^2-1)^{3/2}}{9} + c //$$

$$3) \int \frac{2x}{(4x^2-1)^{1/2}} dx$$

$$\text{Let } u = 4x^2 - 1$$

$$\frac{du}{dx} = 8x \quad dx = \frac{du}{8x}$$

$$\Rightarrow \int \frac{2x}{4^{1/2}} dx = \int \frac{2x}{u^{1/2}} \frac{du}{8x}$$

$$= \frac{1}{4} \int u^{-1/2} du = \frac{1}{4} \int \frac{u^{1/2}}{1/2} + C$$

$$= \frac{u^{1/2}}{2} + C \Rightarrow \frac{(4x^2-1)^{1/2}}{2} + C$$

$$\text{OR } \frac{\sqrt{4x^2-1}}{2} + C$$