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Mat 104

1 $y = \frac{(x+1)^2 (x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}$

$\ln y = [\ln(x+1)^2 + \ln(x-2)^{1/2}] - \ln(2x-1) - \ln(x-3)^{4/3}$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{(x+1)^2} \times 2(x+1) + \frac{1}{(x-2)^{1/2}} \times \frac{(x-2)^{-1/2}}{2} - \frac{1}{2x-1} \times 2 - \frac{1}{x-3} \cdot \frac{4}{3}(x-3)^{1/3}$

$\frac{1}{y}\frac{dy}{dx} = \frac{2(x+1)}{(x+1)^2} + \frac{1}{\sqrt{x-2}} \times \frac{1}{2\sqrt{x-2}} - \frac{2}{2x-1} - \frac{4}{3}\frac{(x-3)^{1/3}}{(x-3)^{4/3}}$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{4}{3(x-3)}$

$\frac{dy}{dx} = y\left[\frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{4}{3(x-3)}\right]$

$\frac{dy}{dx} = \frac{(x+1)^2 (x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}\left[\frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{4}{3(x-3)}\right]$

2 $y = \frac{3e^K \sin 2K}{K^{5/2}}$

$\ln y = \ln(3e^K) + \ln(\sin 2K) - \ln(K^{5/2})$

$\frac{1}{y}\frac{dy}{dK} = \frac{1}{3e^K} \cdot 3e^K + \frac{1}{\sin 2K} 2\cos 2K - \frac{1}{K^{5/2}} \cdot \frac{5}{2} K^{3/2}$

$\frac{1}{y}\frac{dy}{dK} = 1 + \frac{2\cos 2K}{\sin 2K} - \frac{5}{2} K^{3/2 - 5/2}$

$\frac{dy}{dK} = y\left[1 + \frac{2\cos 2K}{\sin 2K} - \frac{5}{2}K^{-1}\right]$

$\frac{dy}{dK} = \frac{3e^K \sin 2K}{K^{5/2}}\left[1 + \frac{2\cos 2K}{\sin 2K} - \frac{5}{2K}\right]$

15 $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

$u = \sqrt{4x^2-1}$

$u^2 = 4x^2 - 1$

$4x^2 = u^2 + 1$

$x^2 = \frac{u^2+1}{4}$

$x = \sqrt{\frac{u^2+1}{4}}$

$\frac{dx}{du} = \frac{1}{2}\left(\frac{u^2+1}{4}\right)^{-1/2} \cdot \frac{u}{2}$

$dx = \frac{1}{2}\left(\frac{u^2+1}{4}\right)^{-1/2} \cdot \frac{u\,du}{2}$

$dx = \frac{u\,du}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$

$\int \frac{2}{u}\left(\frac{u^2+1}{4}\right)^{1/2} \cdot \frac{4u\,du}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$

$\int \left(\frac{u^2+1}{4}\right)^{1/2 - 1/2} \cdot \frac{du}{2}$

$\frac{1}{2}\int du$

$\frac{1}{2}[u] + C$

$\frac{u}{2} + C$

$= \frac{\sqrt{4x^2-1}}{2} + C$

(3) $\int 4\sec^2(3m+1)\,dm$

$4\int \sec^2(3m+1)\,dm$ Let $u = 3m+1$

$\frac{du}{dm} = 3$

$du = 3\,dm$

$dm = \frac{du}{3}$

$4\int \sec^2(u)\frac{du}{3}$

$\frac{4}{3}\int \sec^2(u)\,du$

$\frac{4}{3}\tan u + C$

$= \frac{4}{3}\tan(3m+1) + C$

(4) $\int 2t\,(3t^2-1)^{1/2}\,dt$

$u = \sqrt{3t^2-1}$

$u^2 = 3t^2 - 1$

$3t^2 = u^2 + 1$

$t^2 = \frac{u^2+1}{3}$

$t = \sqrt{\frac{u^2+1}{3}}$

$\frac{dt}{du} = \frac{1}{2}\left(\frac{u^2+1}{3}\right)^{-1/2} \cdot \frac{2u}{3}$

$\frac{dt}{du} = \frac{u}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$dt = \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$\int 2\left(\frac{u^2+1}{3}\right)^{1/2} \cdot u \cdot \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$\int \frac{2}{3}\left(\frac{u^2+1}{3}\right)^{1/2-1/2} \times u^2\,du$

$\frac{2}{3}\int u^2\,du$

$\frac{2}{3}\left[\frac{u^3}{3}\right] + C$

$\frac{2u^3}{9} + C = \frac{2(3t^2-1)^{3/2}}{9} + C$