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Matric No:- 19/ENG05/060

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Assignment

(1) Find the limit of the function $(x - \cos x)/x$ as $x \to 0$

solution

(a) using L'Hopital's rule, we have

$$\lim_{x \to 0} \left\{ \frac{x - \cos x}{x} \right\} = \lim_{x \to 0} \left\{ \frac{1 + \sin x}{1} \right\}$$

$$= \lim_{x \to 0} \{1 + \sin x\} = 1 + 0$$

- The limit of the function $((x - \cos x)/x)$ as $x \to 0 = 1$

(2) If $y = -3 \tan 7x \, e^{3x}$ find dy/dx

$$\frac{dy}{dx} = y \left[\frac{1}{u} \frac{du}{dx} + \frac{1}{v} \frac{dv}{dx} + \frac{1}{w} \frac{dw}{dx} \right]$$

$$\frac{dy}{dx} = y \left[\frac{-1}{3}(0) + \frac{1}{\tan 7x} [7 \sec^2 7x] + \frac{1}{e^{3x}} [3 e^{3x}] \right]$$

$$\frac{dy}{dx} = y \left[0 + \frac{7 \sec^2 7x}{\tan 7x} + 3 \right]$$

$$\frac{dy}{dx} = -3 \tan 7x \, e^{3x} \left[3 + \frac{7 \sec^2 7x}{\tan 7x} \right]$$

(3) If $y = \cos 3x$, Find dy/dx from the first principle

Solution

$$y = \cos 3x$$

$$y + \Delta y = \cos 3(x + \Delta x)$$

$$\Delta y = \cos 3(x+\Delta x) - \cos 3x \quad \text{---- (1)}$$

recall $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$ ---

Compare equ (I) and (2)

$$\frac{A+B}{2} = \frac{3(x+\Delta x)+3x}{2}$$

$$= \frac{3x + 3\Delta x + 3x}{2}$$

$$= \frac{3x + 3\Delta x}{2}$$

$$\frac{A-B}{2} = \frac{3x + 3\Delta x - 3x}{2}$$

$$= \frac{3\Delta x}{2}$$

$$\Delta y = -2\sin\left(\frac{3x+3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)$$

Divide Both sides by Δx

$$\frac{\Delta y}{\Delta x} = \frac{-2\sin\left(\frac{3x+3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)}{\Delta x}$$

Divide RHS by 2

$$\frac{\Delta y}{\Delta x} = \frac{-2\sin\left(\frac{3x+3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)}{\frac{\Delta x}{2}}$$

$$\frac{\Delta y}{\Delta x} = -\sin\left(\frac{3x+3\Delta x}{2}\right)\frac{\sin\left(\frac{3\Delta x}{2}\right)}{\frac{\Delta x}{2}} \times 3$$

taking the limit as Δx tends to 0

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = -\sin\left(\frac{3x+3(0)}{2}\right)\lim_{\Delta x \to 0}\frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}} \times 3$$

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = -\sin 3x \times 3 \quad \left[\text{since } \lim_{\Delta x \to 0}\frac{\sin\theta}{\theta} = 1\right]$$

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = -3\sin 3x$$

(4) Given $F(x) = 2x^3 - 7x$ and $g(x) = -3x$, Find $(F-g)(5)$

Solution

$F - g(x) = 2x^3 - 7x - (-3x)$

$F - g(x) = 2x^3 - 7x + 3x$

$F - g(x) = 2x^3 - 4x$

$F - g(5) = 2(5)^3 - 4(5)$

$= 250 - 20$

$F - g(5) = 230$

(5) Find $F \circ g(x)$ if $F(x) = 4x^2 + 2$ and $g(x) = 2x + 3$

Solution

$F \circ g(x) = 4(2x+3)^2 + 2$

$= 4(2x+3)(2x+3) + 2$

$= 4(4x^2 + 6x + 6x + 9) + 2$

$= 4(4x^2 + 12x + 9) + 2$

$= 16x^2 + 48x + 36 + 2$

$F \circ g(x) = 16x^2 + 48x + 38$

(6) Find the gradient of $x^2 + 2xy + y^2 =$

$$\frac{d}{dx}(x^2) + 2\left[\frac{d}{dx} x \cdot y + \frac{dy}{dx} \cdot x + 2y \frac{dy}{dx}\right] = 0$$

$$2x + 2y + \frac{x\,dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 2y$$

$$(x + 2y)\frac{dy}{dx} = -2x - 2y$$

$$\frac{dy}{dx} = \frac{-2x - 2y}{x + 2y}$$

(7) Find the first derivative of the function

$y = x^2 \cos x$

$u = x^2, \quad \frac{du}{dx} = 2x$

$v = \cos x, \quad \frac{dv}{dx} = -\sin x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = x^2(-\sin x) + \cos x(2x)$$

$$\frac{dy}{dx} = -x^2 \sin x + 2x \cos x$$