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19/ENG/04/009
Electrical Electronic Engineering
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1 $y = \frac{(x+1)^2 (x-2)^{\frac{1}{2}}}{(2x-1)(x+3)^{3/2}}$

$\ln y = \ln[(x+1)^2] + \ln(\sqrt{x-2}) - \ln(2x-1) - \ln[(x+3)^{3/2}]$

$\left(\frac{1}{y}\right)\frac{dy}{dx} = \frac{1}{(x+1)^2} \cdot 2(x+1) + \frac{1}{\sqrt{x-2}} \cdot \frac{1}{2}(x-2)^{-\frac{1}{2}} - \frac{1}{2x-1} \cdot 2 - \frac{1}{(x+3)^{3/2}} \cdot \frac{3}{2}(x+3)^{\frac{1}{2}}$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{2}{(x+1)} + \frac{1}{2(\sqrt{x-2})(\sqrt{x-2})} - \frac{2}{2x-1} - \frac{3}{2}(x+3)^{\frac{1}{2}-\frac{3}{2}}$

$\frac{dy}{dx} = y\left[\frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{3}{2(x+3)}\right]$

$\frac{dy}{dx} = \frac{(x+1)^2 (x-2)^{\frac{1}{2}}}{(2x-1)(x+3)^{\frac{3}{2}}}\left[\frac{2}{x+1} + \frac{1}{2x-4} - \frac{2}{2x-1} - \frac{3}{2x+6}\right]$

2 $y = \frac{3e^k \sin 2k}{k^{5/2}}$

$\ln y = \ln(3e^k) + \ln(\sin 2k) - \ln(k^{5/2})$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{3e^k} \cdot 3e^k + \frac{1}{\sin 2k} \cdot 2\cos 2k - \frac{1}{k^{5/2}} \cdot \frac{5}{2} k^{3/2}$

$\frac{1}{y} \cdot \frac{dy}{dx} = 1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2} x^{\frac{3}{2}-\frac{5}{2}}$

$\frac{dy}{dx} = y\left[1 + \frac{2\cos 2k}{\sin 2k} - \frac{5k^{-1}}{2}\right]$

$\frac{dy}{dx} = \frac{3e^k \sin 2k}{k^{5/2}}\left[1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2k}\right]$

3) $\int 4\sec^2(3m+1)\,dm$

Soln

$4\int \sec^2(3m+1)\,dm$

Let $u = 3m+1$

$\frac{du}{dm} = 3$

$du = 3dm$, $dm = \frac{du}{3}$

$4\int \sec^2(u)\frac{du}{3}$

$\frac{4}{3}\int \sec^2(u)\,du$

$\frac{4}{3}\tan u + c$

$= \frac{4}{3}\tan(3m+1)+c$

4) $\int 2t(3t^2-1)^{1/2}\,dt$

Soln

Let $u = \sqrt{3t^2-1}$

$u^2 = 3t^2-1$

$3t^2 = u^2+1$

$t^2 = \frac{u^2+1}{3}$

$t = \sqrt{\frac{u^2+1}{3}}$

$\frac{dt}{du} = \frac{1}{2}\left(\frac{u^2+1}{3}\right)^{-1/2} \cdot \frac{2u}{3}$

$\frac{dt}{du} = \frac{u}{3}\sqrt{\frac{3}{u^2+1}}$

$\frac{dt}{du} = \frac{u}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$dt = \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$$\int 2\left(\frac{v^2+1}{3}\right)^{\frac{1}{2}} \cdot v \cdot \frac{v\,dv}{3}\left(\frac{v^2+1}{3}\right)^{-\frac{1}{2}}$$

$$= \frac{2}{3}\int v^2\left(\frac{v^2+1}{3}\right)^{\frac{1}{2}-\frac{1}{2}} dv$$

$$= \frac{2}{3}\int v^2 dv$$

$$= \frac{2}{3}\left[\frac{v^3}{3}\right] + c$$

$$= \frac{2v^3}{9} + c$$

$$= \frac{2(3t^2-1)^{3/2}}{9} + c$$

5) $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

Soln

Let $v = \sqrt{4x^2-1}$

$v^2 = 4x^2-1$

$4x^2 = v^2+1$

$x^2 = \frac{v^2+1}{4}$

$x = \sqrt{\frac{v^2+1}{4}}$

$\frac{dx}{dv} = \frac{1}{2}\left(\frac{v^2+1}{4}\right)^{-\frac{1}{2}} \cdot \frac{v}{2}$

$\frac{dx}{dv} = \frac{v}{4}\left(\frac{v^2+1}{4}\right)^{-\frac{1}{2}}$

$dx = \frac{v\,dv}{4}\left(\frac{v^2+1}{4}\right)^{-\frac{1}{2}}$

$\int \frac{\cancel{2}\left(\frac{v^2+1}{4}\right)^{\frac{1}{2}}}{\cancel{v}} \cdot \frac{\cancel{v}\,dv}{\cancel{4}_2}\left(\frac{v^2+1}{4}\right)^{-\frac{1}{2}}$

$\frac{1}{2}\int\left(\frac{v^2+1}{4}\right)^{\frac{1}{2}-\frac{1}{2}} dv$

$\frac{1}{2}\int dv$

$$= \frac{u}{2} + C$$

$$= \cancel{\sqrt{4x^2 -}}$$

$$= \frac{\sqrt{4x^2-1}}{2} + C$$