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Computer Science.
MAT 104.

Assignment

1) Find the limit of the function $\left(\frac{x - \cos x}{x}\right)$ as $x \to 0$.

Soln.

$$\lim_{x \to 0} \frac{(x - \cos x)}{x}$$

By L'Hopital's rule;

$$\Rightarrow \lim_{x \to 0} \frac{[1 - (-\sin x)]}{1}$$

$$= \lim_{x \to 0} \frac{(1 + \sin x)}{1} = \frac{[1 + \sin(0)]}{1} = \frac{1+0}{1} = 1.$$

2) If $y = -3\tan 7x\, e^{3x}$, find dy/dx.

$u = -3\tan 7x \quad v = e^{3x}$

$\frac{du}{dx} = -21\sec^2 7x \quad \frac{dv}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = (-3\tan 7x)(3e^{3x}) + e^{3x}(-21\sec^2 7x)$$

$$\frac{dy}{dx} = [(-3\tan 7x)(3e^{3x}) + e^{3x}(-21\sec^2 7x)]$$

$$\frac{dy}{dx} = [3e^{3x}(-3\tan 7x) + e^{3x}(-21\sec^2 7x)]$$

3.) If $y = \cos 3x$, find dy/dx from the first principle.

Using the formula

$$\frac{dy}{dx} \lim_{\Delta x \to 0} \frac{f[x+\Delta x] - f(x)}{\Delta x}$$

$$= \frac{\cos[3(x+\Delta x)] + \cos 3x}{\Delta x}$$

$$= \frac{\cos[3x + 3\Delta x] - \cos 3x}{\Delta x}$$

Recall, $\cos(A+B) = \cos A \cos B - \sin A \sin B$.

$$= \frac{[\cos 3x \cos(3\Delta x) - \sin 3x \sin(3\Delta x)] - \cos 3x}{\Delta x}$$

Collect like terms.

$$= \left[\left(\frac{\cos 3x \cos(3\Delta x) - \cos 3x}{\Delta x}\right)\right] - \left[\left(\frac{\sin 3x \sin(3\Delta x)}{\Delta x}\right)\right]$$

$$= \lim_{\Delta x \to 0} \left[\cos 3x\left(\frac{\cos(3\Delta x) - 1}{\Delta x}\right)\right] - \left[\left(\sin 3x\left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right)\right]$$

$= \lim_{\Delta x \to 0}\left[\cos 3x\left(\frac{\cos(3\Delta x)-1}{\Delta x} \times \frac{\cos(3\Delta x+1)}{\cos(3\Delta x+1)}\right)\right] - \left[\sin 3x\left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$

$= \lim_{\Delta x \to 0}\left[\cos 3x\left(\frac{\cos^2(3\Delta x)-1}{\cos(3\Delta x+1)\Delta x}\right)\right] - \left[\sin 3x\left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$

From the identity $\sin^2(\theta) + \cos^2(\theta) = 1$

$\cos^2(3\Delta x) - 1 = -\sin^2(3\Delta x)$

$= \lim_{\Delta x \to 0}\left[-\cos 3x\left(\frac{\sin^2(3\Delta x)}{\cos(3\Delta x+1)\Delta x}\right)\right] - \left[\sin 3x\left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$

$= \lim_{\Delta x \to 0}\left[\frac{-\cos 3x \sin^2(3\Delta x)}{\cos(3\Delta x+1)\Delta x} - \sin 3x\right]\left[\frac{\sin(3\Delta x)}{\Delta x}\right]$

Since the lim. of a product is the product of the limits,

$\therefore \left(\lim_{\Delta x \to 0}\left[\frac{-\cos 3x \sin^2(3\Delta x)}{\cos(3\Delta x+1)\Delta x} - \sin 3x\right]\right) \cdot \left(\lim_{\Delta x \to 0}\left[\frac{\sin(3\Delta x)}{\Delta x}\right]\right)$

$= \frac{-\cos 3x \sin^2(3(0))}{\cos(0+1)0} - \sin 3x$ as the first limit which

simplifies to $-\sin 3x$, $-\sin 3x \lim_{\Delta x \to 0} \frac{\sin(3\Delta x)}{\Delta x}$

Recall: $\lim_{\Delta x \to 0} \frac{\sin x}{x} = 1$ $\therefore$ $\lim_{\Delta x \to 0} \frac{\sin(3\Delta x)}{\Delta x} = 3$

$\therefore (-\sin 3x)(3) = -3\sin 3x$.

4.) Given that $f(x) = 2x^2 - 7x$ and $g(x) = -3x$, find $(f-g)(5)$.

$(f-g)(5) = [(2(5)^2 - 7(5)) - (-3(5))]$

$(f-g)(5) = [250 - 35] + 15$.

$(f-g)(5) = 230$

5.) Find $f \circ g(x)$ if: $f(x) = 4x^2 + 2$, $g(x) = 2x + 3$.

$f \circ g(x) = 4(2x+3)^2 + 2$

$= 4[4x^2 + 12x + 9] + 2$.

$= 16x^2 + 48x + 36 + 2$.

$= 16x^2 + 48x + 38$.

6.) Find the gradient of $x^2 + 2xy + y^2 = 1020$.

$x^2 + 2xy + y^2 = 1020$

$2x\,dx/dx + 2x\,dy/dx + 2y\,dx/dx + 2y\,dy/dx = 0$.

$2x + 2x\,dy/dx + 2y + 2y\,dy/dx = 0$.

$2x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 2y$