MAT 104

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Computer Science

(1) Find the limit of the function $(x - \cos x)/x$ as $x \to 0$

Solution

$$\lim_{x \to 0} \frac{(x - \cos x)}{x}$$

By l'opitals rule we have

$$= \lim_{x \to 0} \frac{(1 - (-\sin x))}{1}$$

$$= \lim_{x \to 0} \frac{(1 + \sin x)}{1} = \frac{1+0}{1} = \frac{1}{1} = 1$$

(2) If $y = -3\tan 7x e^{3x}$, find dy/dx.

$U = -3\tan 7x \qquad V = e^{3x}$

$du/dx = -21\sec^2 7x \qquad dv/dx = 3e^{3x}$

$dy/dx = (-3\tan 7x)(3e^{3x}) + e^{3x}(-21\sec^2 7x)$

$dy/dx = [(-3\tan 7x)(3e^{3x}) + e^{3x}(-21\sec^2 7x)]$

$dy/dx = [3e^{3x}(-3\tan 7x) + e^{3x}(-21\sec^2 7x)]$

(6) If $y = \cos 3x$, find dy/dx from the first principle using the formular

$$\frac{dy}{dx} = \lim_{x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{\cos[3(x+\Delta x)] - \cos 3x}{\Delta x}$$

$$= \frac{\cos[3x + 3\Delta x] - \cos 3x}{\Delta x}$$

Recall

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{[\cos 3x \cos(3\Delta x) - \sin 3x \sin(3\Delta x)] - \cos 3x}{\Delta x}$$

$$= \left[\frac{\cos 3x \cos(3\Delta x) - \cos 3x}{\Delta x}\right] - \left[\frac{\sin 3x \sin 3\Delta x}{\Delta x}\right]$$

$$= \lim_{x \to 0}\left[\cos 3x \left(\frac{\cos(3\Delta x) - 1}{\Delta x}\right)\right] - \left[\sin 3x \left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$$

$$= \lim_{x \to 0}\left[\cos 3x \left(\frac{\cos(3\Delta x - 1) \times \cos(3\Delta x + 1)}{\Delta x \cos(3\Delta x + 1)}\right)\right] -$$

$$\left[\sin 3x \left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$$

$$\lim_{\Delta x \to 0}\left[\cos 3x \left(\frac{\cos^2(3\Delta x) - 1}{\cos(3\Delta x + 1)\Delta x}\right)\right] - \left[\sin 3x \left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$$

From the identity $\sin^2(\theta) + \cos^2(\theta) = 1$

$$\cos^2(3\Delta x) - 1 = -\sin^2(3\Delta x)$$

$$= \lim_{\Delta x \to 0}\left[\cos 3x \left(\frac{\sin^2(3\Delta x)}{\cos(3\Delta x + 1)\Delta x}\right)\right] - \left[\sin 3x \left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$$

Using (F) $\lim_{\Delta x \to 0}\left[\frac{-\cos 3x \sin^2(3\Delta x) - \sin 3x}{\cos(3\Delta x+1)\Delta x}\right]\left[\frac{\sin(3\Delta x)}{\Delta x}\right]$

Since the limit of Product is the Products of limit

$= \left(\lim_{\Delta x \to 0}\left[\frac{-\cos 3x \sin^2(3\Delta x) - \sin 3x}{\cos(3\Delta x+1)\Delta x}\right]\right)\cdot\left(\lim_{\Delta x \to 0}\left[\frac{\sin(3\Delta x)}{\Delta x}\right]\right)$

As $\Delta x \to 0$, the first limit goes to $\frac{-\cos 3x \sin^2(30) - \sin 3x}{\cos(0+1)0}$

Which simplifies to $-\sin 3x$. So it can be written as

$-\sin 3x \lim_{x\to 0} \frac{\sin(3\Delta x)}{\Delta x}$

Recall $\lim_{\Delta x \to 0} \frac{\sin x}{x} = 1$ therefore $\lim_{\Delta x \to 0}\frac{\sin(3\Delta x)}{\Delta x} = 3$

The derivative is $(-\sin 3x)(3)$

$= -3\sin 3x$

(4) Given that $f(x) 2x^3 - 7x$ and $g(x) = -3x$, find $(f-g)(5)$

Solution

$(f-g)(5) = f(5) - g(5)$

$(f-g)(5) = [2(5)^3 - 7(5)] - [-3(5)]$

$(f-g)(5) = [2(125) - 35] - [-15]$

$(f-g)(5 = [250 - 35] + 15$

$= 215 + 15$

$= 230$

(5) Find $f\circ g(x)$ if

$f(x) = 4x^2 + 2$

$g(x) = 2x + 3$

Solution

$f\circ g(x) = 4(2x+3)^2 + 2$

$4(2x+3)(2x+3) + 2$

$= 4(4n^2 + 6n + 6n + 9) + 2$

$= 4(4n^2 + 12n + 9) + 2$

$= 16n^2 + 48n + 36 + 2$

$= 16n^2 + 48n + 38$

6) Find the gradient of $n^2 + 2ny + y^2 = 1020$

Solution

$n^2 + 2ny + y^2 = 1020$

$2n \, dn/dn + 2n \, dy/dn + 2y \, dn/dn + 2y \, dy/dn$

$2n + 2n \, dy/dn + 2y + 2y \, dy/dn = 0$

$2n \, dy/dn + 2y \, dy/dn = -2n - 2y$

$dy/dn \, (2n + 2y) = -2n - 2y$

$$dy/dn = \frac{-2n - 2y}{2n + 2y}$$

7. Find the first derivative of the function $y = n^2 \cos n$

Solution

$y = n^2 \cos n$

$u = n^2$, $v = \cos n$

$\frac{du}{dn} = 2n$, $\frac{dv}{dn} = -\sin n$

Using Product rule

$$dy/dn = u\frac{dv}{dn} + v\frac{du}{dn}$$

$dy/dn = (n^2)(-\sin n) + (2n)(\cos n)$

$= -n^2 \sin n + 2n \cos n$

$= [2n \cos n - n^2 \sin n]$