NAME: ALEGBELEYE OLUWATOSIN OLUWAPELUMI
DEPARTMENT: COMPUTER SCIENCE
MATRIC NO: 19/SC101/015

ASSIGNMENT

1. Find the limit of the function $(x - \cos x)/x$ as $x \to 0$.

solution

$$\lim_{x \to 0} \frac{(x - \cos x)}{x}$$

By L'Hopital's rule, we have

$$= \lim_{x \to 0} \frac{(1 - (-\sin x))}{1}$$

$$= \lim_{x \to 0} \frac{(1 + \sin x)}{1} = \frac{(1 + \sin(0))}{1} = \frac{1+0}{1} = \frac{1}{1} = \frac{1}{4}$$

2. If $y = -3\tan 7x\, e^{3x}$, find dy/dx

solution

$u = -3\tan 7x \qquad v = e^{3x}$

$\frac{du}{dx} = -21\sec^2 7x \qquad \frac{dv}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = (-3\tan 7x)(3e^{3x}) + e^{3x}(-21\sec^2 7x)$$

$$\frac{dy}{dx} = [(-3\tan 7x)(3e^{3x}) + e^{3x}(-21\sec^2 7x)]$$

$$\frac{dy}{dx} = [3e^{3x}(-3\tan 7x) + e^{3x}(-21\sec^2 7x)]$$

(3.) If $y = \cos 3x$, Find dy/dx from the first principle using the formular

$$\frac{dy}{dx} \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{\cos[3(x+\Delta x)] - \cos 3x}{\Delta x}$$

$$= \frac{\cos[3x + 3\Delta x] - \cos 3x}{\Delta x}$$

Recall,

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$= \frac{[\cos 3x \cos(3\Delta x) - \sin 3x \sin(3\Delta x)] - \cos 3x}{\Delta x}$$

Collect like terms

$$= \left[\left(\frac{\cos 3x \cos(3\Delta x) - \cos 3x}{\Delta x}\right)\right] - \left[\left(\frac{\sin 3x \sin(3\Delta x)}{\Delta x}\right)\right]$$

$$= \lim_{\Delta x \to 0}\left[\cos 3x\left(\frac{\cos(3\Delta x) - 1}{\Delta x}\right)\right] - \left[\sin 3x\left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$$

$$= \lim_{\Delta x \to 0}\left[\cos 3x\left(\frac{\cos(3\Delta x) - 1}{\Delta x} \times \frac{\cos(3\Delta x + 1)}{\cos(3\Delta x + 1)}\right)\right] - \left[\sin 3x\left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$$

$$= \lim_{\Delta x \to 0}\left[\cos 3x\left(\frac{\cos^2(3\Delta x) - 1}{\cos(3\Delta x + 1)\Delta x}\right)\right] - \left[\sin 3x\left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$$

from the identity $\sin^2(\theta) + \cos^2(\theta) = 1$

$\cos^2(3\Delta x) - 1 = -\sin^2(3\Delta x)$

$$= \lim_{\Delta x \to 0}\left[-\cos 3x\left(\frac{\sin^2(3\Delta x)}{\cos(3\Delta x + 1)\Delta x}\right)\right] - \left[\sin 3x\left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$$

$$= \lim_{\Delta x \to 0} \left[\frac{-\cos 3x \sin^2(3\Delta x) - \sin 3x}{\cos(3\Delta x + 1)\Delta x}\right]\left[\frac{\sin(3\Delta x)}{\Delta x}\right]$$

Since the limit of a product is the product of the limits, we have

$$= \left(\lim_{\Delta x \to 0}\left[\frac{-\cos 3x \sin^2(3\Delta x) - \sin 3x}{\cos(3\Delta x + 1)\Delta x}\right]\right) \cdot \left(\lim_{\Delta x \to 0}\left[\frac{\sin(3\Delta x)}{\Delta x}\right]\right)$$

As $\Delta x \to 0$, the first limit goes to $\frac{-\cos 3x \sin^2(3(0))}{\cos(0+1)0} - \sin 3x$,

which simplifies to $-\sin 3x$; so the derivative can be re-written as

$$-\sin 3x \lim_{\Delta x \to 0} \frac{\sin(3\Delta x)}{\Delta x}$$

Recall $\cdot \lim_{x \to 0} \frac{\sin x}{x} = 1$, therefore $\lim_{\Delta x \to 0} \frac{\sin(3\Delta x)}{\Delta x} = 3$

So the final form of the derivative is $(-\sin 3x)(3)$

$$= -3\sin 3x$$

4) Given that $f(x) = 2x^3 - 7x$ and $g(x) = -3x$, find $(f-g)(5)$

Solution

$(f-g)(5) = f(5) - g(5)$

$(f-g)(5) = [2(5)^3 - 7(5)] - [-3(5)]$

$(f-g)(5) = [2(125) - 35] - [-15]$

$(f-g)(5) = [250 - 35] + 15$

$(f-g)(5) = 215 + 15$

$(f-g)(5) = 230$

5. Find $fog(x)$ if

$f(x) = 4x^2+2$

$g(x) = 2x+3$

Solution

$fog(x) = 4(2x+3)^2+2$

$= 4(2x+3)(2x+3)+2$

$= 4(4x^2+6x+6x+9)+2$

$= 4(4x^2+12x+9)+2$

$= 16x^2+48x+36+2$

$= 16x^2+48x+38$

6. Find the gradient of $x^2+2xy+y^2=1020$.

Solution

$x^2+2xy+y^2=1020$

$2x\,\cancel{dx}/\cancel{dx} + 2x\,dy/dx + 2y\,\cancel{dx}/\cancel{dx} + 2y\,dy/dx = 0$

$2x+2x\,dy/dx+2y+2y\,dy/dx = 0$

$$2x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x-2y$$

$$\frac{dy}{dx}\left(2x+2y\right) = -2x-2y$$

$$\frac{dy}{dx} = \frac{-2x-2y}{2x+2y}$$

7. Find the first derivative of the function $y = x^2\cos x$

solution

$y = x^2\cos x$

$u = x^2$, $v = \cos x$

$$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = -\sin x$$

using product rule

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = (x^2)(-\sin x)+(2x)(\cos x)$$

$= -x^2\sin x + 2x\cos x$

$= [2x\cos x - x^2\sin x]$