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DEPT: MECHATRONICS ENGINEERING
MATRIC NO: 19/ENG05/020

1. Find $\lim_{x \to 0} \left\{ \frac{x - \cos x}{x} \right\}$

Solution

$$\lim_{x \to 0} \left\{ \frac{x - \cos x}{x} \right\} = \lim_{x \to 0} \left\{ \frac{x}{x} - \frac{\cos x}{x} \right\}$$

$$= \lim_{x \to 0} \left\{ \frac{1 - (-\sin x)}{1} \right\}$$

$$= \lim_{x \to 0} \left\{ \frac{1 + \sin x}{1} \right\}$$

$$= \frac{1 + \sin 0}{1} = 1 + 0 = 1$$

$$\therefore \lim_{x \to 0} \left\{ \frac{x - \cos x}{x} \right\} = 1$$

2. $y = -3 \tan 7x e^{3x}$ find $\frac{dy}{dx}$

Solution

$u = -3 \tan 7x \qquad v = e^{3x}$

$du = -21 \sec^2 7x \qquad dv = 3e^{3x}$

$$\frac{dy}{dx} = U\frac{dv}{dx} + V\,du/dx$$

$$\frac{dy}{dx} = -3\tan 7x\,(3e^{3x}) + e^{3x}(-21\sec^2 7x)$$

$$\frac{dy}{dx} = -3\tan 7x\,(3e^{3x}) - 21\sec^2 7x\,(e^{3x})$$

$$\frac{dy}{dx} = -9e^{3x}\tan 7x - 21e^{3x}\sec^2 7x$$

$y = \cos x$, find $\frac{dy}{dx}$ from 1st principle

Solution

$y = \cos 3x$

$y + \Delta y = \cos 3(x + \Delta x)$

$y + \delta y = \cos 3(x + \delta x)$

$\delta y = \cos 3(x + \delta x) - y$

$\delta y = \cos 3(x + \delta x) - \cos 3x$

ie where $y = \cos 3x$

and $A = 3(x + \delta x)$ $\quad B = 3x$

$= 3x + 3\delta x$

Recall $\cos A - \cos B = -2 \sin \frac{A+B}{2} \cdot \sin \frac{A-B}{2}$

$$\therefore \delta y = -2 \sin\left(\frac{3x+3\delta x+3x}{2}\right) \cdot \sin\left(\frac{3x+3\delta x-3x}{2}\right)$$

$$\delta y = -2 \sin\left(\frac{6x+3\delta x}{2}\right) \cdot \sin \frac{3\delta x}{2}$$

$$\delta y = -2 \sin\left(3x + 3\delta x/2\right) \cdot \sin 3\delta x/2$$

$$\frac{\delta y}{\delta x} = \frac{-2 \sin\left(3x + 3\delta x/2\right) \cdot \sin(3\delta x/2)}{\delta x}$$

$$\frac{\delta y}{\delta x} = \frac{-2 \sin\left(3x + 3\delta x/2\right) \cdot \sin(3\delta x/2) \div 2}{\delta x/2}$$

$$\frac{\delta y}{\delta x} = \frac{-\sin\left(3x + \frac{3\delta x}{2}\right) \cdot \sin \frac{3\delta x}{2}}{\delta x/2}$$

$$\frac{\delta y}{\delta x} = -\sin\left(3x + \frac{3\delta x}{2}\right) \cdot \frac{\sin \frac{3\delta x}{2}}{\delta x/2}$$

Recall $\frac{\sin x}{x} = 1$

$$\lim_{x \to 0} = \left(\frac{\sin 3\,\delta x/2}{\delta x/2}\right) = 3$$

$$\lim_{x \to 0} = -\sin\left(3x + \frac{3\delta x}{2}\right) = -\sin(3x$$

$$= -\sin 3x$$

$$\frac{dy}{dx} = -\sin 3x \cdot 3$$

$$\frac{dy}{dx} = -3\sin 3x$$

4. $f(x) = 2x^3 - 7x$

$g(x) = -3x$

$\therefore (f-g)(5) = ?$

Solution

$f(x) = 2x^3 - 7x \qquad g(x) = 3x$

$f - g = (2x^3 - 7x) - (-3x)$

$f - g = 2x^3 - 7x + 3x = 2x^3 - 4x$

$(f-g)(5) = 2(5^3) - 4(5)$

$(f-g)(5) = 2(125) - 20 = 250 - 20$

$((f-g)(5)) = 230$

5 Find $fog(x)$ if $f(x) = 4x^2 + 2$

$g(x) = 2x + 3$

Solution

$f(x) = 4x^2 + 2 \qquad g(x) = 2x + 3$

$fog(x) = 4(2x+3)^2 + 2$

$fog(x) = 4(4x^2 + 12x + 9) + 2$

$fog(x) = 16x^2 + 48x + 36 + 2$

$fog(x) = 16x^2 + 48x + 38$

6. Find the gradient of $x^2 + 2xy + y^2 = 1020$

Solution

$m = \frac{dy}{dx}$ By differentiating and multiplying by m

$2x\frac{dx}{dx} + 2\left(x\frac{dy}{dx} + y\frac{dx}{dx}\right) + 2y\frac{dy}{dx} = 1020\,dx$

$2x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 1020\,dx$

$2x + 2x\,dy/dx + 2y + 2y\,dy/dx = 0$

$2x\frac{dy}{dx} + 2y\frac{dy}{dx} = -(2x+2y)$ DBS$(2x+2y)$

$\frac{dy}{dx}\frac{(2x+2y)}{(2x+2y)} = -\frac{(2x+2y)}{(2x+2y)}$

$$\frac{dy}{dx} = -1$$

7. find the derivative of the function

$$y = x^2 - \cos x$$

Solution

let $u = x^2$

$du/dx = 2x$

$v = \cos x$

$dv/dx = -\sin x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\,du/dx$$

$$\frac{dy}{dx} = x^2(-\sin x) + \cos x(2x)$$

$$\frac{dy}{dx} = -x^2\sin x + 2x\cos x$$