1 Find the limit of the function $(x-\cos x)/x$ as $x \to 0$

Solution

$$\lim_{x \to 0} \frac{(x-\cos x)}{x}$$

$$\lim_{x \to 0} \frac{(1-(-\sin x))}{1}$$

$$= \lim_{x \to 0} \frac{(1+\sin x)}{1} = \frac{1+\sin(0)}{1} = \frac{1}{1} = 1//$$

2 If $y = -3\tan 7x e^{3x}$. Find dy/dx

Solution

$u = -3 \quad v = \tan 7x \quad w = e^{3x}$

$du/dx = 0 \quad dv/dx = 7\sec^2 7x \quad dw/dx = 3e^{3x}$

$$\frac{dy}{dx} = y\left[\frac{-1}{3}(0) + \frac{1}{\tan 7x}(7\sec^2 7x) + \frac{1}{e^{3x}}(3e^{3x})\right]$$

$$\frac{dy}{dx} = [y]\left[0 + \frac{7\sec^2 7x}{\tan 7x} + 3\right]$$

$$\frac{dy}{dx} = \left[\frac{7\sec^2 7x}{\tan 7x} + 3\right]\left[-3\tan 7x e^{3x}\right]$$

3 If $y = \cos 3x$. Find dy/dx from first principle

Solution

$y = \cos 3x$

$y + \Delta y = \cos(3x + \Delta x)$

$\Delta y = \cos(3x + \Delta x) - \cos 3x$ — ①

Recall $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

Comparing equation 1 & 2

$A = 3x + \Delta x$ $\qquad B = 3x$

$\frac{A+B}{2} = \frac{3x + \Delta x + 3}{2}$ $\qquad \frac{6x + \Delta x}{2} = 6x + \frac{\Delta x}{2}$

and

$\frac{A-B}{2} = \frac{3x + \Delta x - 3x}{2} = \frac{\Delta x}{2}$

Hence

$\Delta y = \cos(3x + \Delta x) - \cos 3x$

$= -2 \sin\left(6x + \frac{\Delta x}{2}\right) \sin \frac{\Delta x}{2}$

Divide through by Δx: $\frac{\Delta y}{\Delta x} = \left[-2 \sin\left[6x + \frac{\Delta x}{2}\right] \left(\sin \frac{\Delta x}{2}\right)\right] \div \Delta$

$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$, $\frac{\sin B}{B} = 1$

$\frac{\Delta y}{\Delta x} = \frac{-2 \sin\left(6x + \frac{\Delta x}{2}\right)\left(\sin \frac{\Delta x}{2}\right) \times \frac{1}{2}}{\Delta x \times \frac{1}{2}}$

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = -2 \sin(6x + 0) \lim_{\Delta x \to 0} \sin\left(\frac{\Delta x}{2}\right) \Big/ \frac{\Delta x}{2}$

Since $\lim_{\Delta x \to 0} \sin\left(\frac{\Delta x}{2}\right) = 1$

$\therefore dy/dx = -2 \sin 6x$

4 $f(x) = 2x^3 - 7x$ and $g(x) = -3x$

$(f-g)(5) = f(5) - g(5)$

$(f-g)(5) = [2(5)^3 - 7(5)][-3(5)]$

$(f-g)(5) = [2(125) - 35] - [-15]$

$(f-g)(5) = 250 - 35 + 15$

$(f-g)(5) = 230$

5 Find $fg(x)$ if $f(x) = 4x^2 + 2$ and $g(x) = 2x + 3$

Solution

$f(g(x)) = 4[2x+3]^2 + 2$

$= 4[4x^2 + 12x + 9] + 2$

$= 16x^2 + 48x + 36 + 2$

$= 16x^2 + 48x + 38$

6 Gradient of $x^2 + 2xy + y^2 = 1020$

$2x\,dy/dx + 2x\,dy/dx + 2y\,dx/dx + 2y\,dy/dx = 0$

$2x + 2x\,dy/dx + 2y + 2y\,dy/dx = 0$

$\left(2x\frac{dy}{dx}\right) + \left(2y\frac{dy}{dx}\right) = -2x - 2y$

$\frac{dy}{dx}(2x + 2y) = -2x - 2y$

$\frac{dy}{dx} = \frac{-2x - 2y}{2x + 2y}$

7 Find first derivative of the function of $y = x^2 \cos x$

$y = x^2 \cos x$

let derivative $= 2x - \sin x$