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MAT 102 Assignment

1.) Find the limit of the function $\lim_{x \to 0} \frac{\sin x}{x}$

$dy/dx = \frac{1+\sin x}{1}$ $x \to 0$

$\frac{1+0}{1}$

$dy/dx = \frac{1}{1}$

2.) If $y = -3\tan 7x e^{3x}$ find $dy/dx$

let $u = -3\tan 7x$ & $v = e^{3x}$

$dy/dx = -21\sec^2 x$ $dv/dx = 3e^{3x}$

$= u\, dv/dx + v\, du/dx$

$= 3\tan 7x(3e^{3x}) + e^{3x}(21\sec^2 x)$

3.) $y = \cos 3x$

Using first principle

$y + \Delta Y = \cos(3x + \Delta x)$

$\Delta Y = \cos(3x + \Delta x) - Y$

$\Delta Y = \cos(3x + \Delta x) - \cos 3x$

Recall $\cos A - \cos B = 2\sin \frac{A+B}{2} \sin \frac{A-B}{2}$

$A = 3x + \Delta x$ ; $B = 3x$

$A+B = \frac{3x + \Delta x + 3x}{2} = \frac{6x + \Delta x}{2} = 3x + \frac{\Delta x}{2}$

$A - B = \frac{3x + \Delta x - 3x}{2} = \frac{\Delta x}{2}$

$\frac{\Delta Y}{\Delta x} = \frac{\sin(3x + \Delta x/2)\ \sin(\Delta x/2) \times 3/2}{\Delta x \times 3/2}$

$$\frac{\Delta y}{\Delta x} = -\sin(3x+0) \lim_{x \to 0} \frac{\sin\left(\frac{3\Delta x}{2}\right)}{\left(\frac{3\Delta x}{2}\right)}$$

Since $\lim_{\Delta x \to 0} \sin \frac{(3\Delta x/2)}{(3\Delta x/2)} = 1$

$$\frac{\Delta y}{\Delta x} = -3\sin 3x$$

4.) $f(x) = 2x^2 - 7x \quad g(x) = -3x$

$(f-g)(5) = f(5) - g(5)$

$= [2(5)^2 - 7(5)] - [-3(5)]$

$= (250 - 35)$

$= 215 + 15$

$= 230$

5.) $f(x) = 4x^2 + 2 \quad g(x) = 2x + 3$

$f \Rightarrow g(x) = 4(2x+3)^2 + 2$

$4(4x^2 + 12x + 9) + 2$

$16x^2 + 48x + 36 + 2$

$16x^2 + 48x + 38$

6.) Find the gradient of $x^2 + 2xy + y^2 = 1020$

$x^2 + 2xy + y^2 = 1020$

$2x + 2x\, dy/dx + 2y + 2y\, dy/dx = 0$

$2x + 2x\, dy/dx + 2y + 2y\, dy/dx = 0$

$2x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2x - 2y$

$2\, dy/dx\,(2x + 2y) = -2x - 2y$

$dy/dx = -\frac{2x - 2y}{2x + 2y}$

7). $y = x^2 \cos x$

$dy/dx = x^2 \cos x$

$u = x^2 \quad v = \cos x$.

$\frac{dy}{dx} = 2x \quad \frac{dv}{dx} = \sin x$

$\frac{dy}{dx} = u\, dv/dx + v\, du/dx$

$= x^2(-\sin x) + \cos x(2x)$

$= -x^2 \sin x + 2x \cos x$