

PINNICK TISE ORITSEBUNDDE

CHEMICAL ENGINEERING

19/ENGG01/013

SERIAL NUMBER : 48

MAT 104 Assignment

Differentiate the following

Question 1

$$y = \frac{[(n+1)^2 (n-2)^{1/2}]}{[(2n-1)(n+3)^{3/2}]}$$

$$\ln y = [2\ln(n+1) + \frac{1}{2}\ln(n-2)] - [\ln(2n-1) + \frac{3}{2}\ln(n+3)]$$

$$\frac{1}{y} \cdot \frac{dy}{dn} = \left(2 \cdot \frac{1}{n+1} \cdot 1 + \frac{1}{2} \cdot \frac{1}{n-2} \cdot 1 \right) - \left(\frac{1}{2n-1} \cdot 2 + \frac{3}{2} \cdot \frac{1}{n+3} \cdot 1 \right)$$

$$\frac{1}{y} \cdot \frac{dy}{dn} = \left(\frac{2}{n+1} + \frac{1}{2(n-2)} \right) - \left(\frac{2}{2n-1} + \frac{3}{2(n+3)} \right)$$

$$\frac{dy}{dn} = y \left[\frac{2}{n+1} + \frac{1}{2(n-2)} - \frac{2}{2n-1} - \frac{3}{2(n+3)} \right]$$

$$\therefore \frac{dy}{dn} = \frac{[(n+1)^2 (n-2)^{1/2}]}{[(2n-1)(n+3)^{3/2}]} \left[\frac{2}{n+1} + \frac{1}{2(n-2)} - \frac{2}{2n-1} - \frac{3}{2(n+3)} \right]$$

Question 2

$$y = \frac{3e^k \sin 2k}{k^{5/2}}$$

$$\ln y = (\ln(3e^k) + \ln(\sin 2k)) - \left(\frac{5}{2} \ln k \right)$$

$$\frac{1}{y} \cdot \frac{dy}{dk} = \frac{1}{3e^k} \cdot 3e^k + \frac{1}{\sin 2k} \cdot 2 \cos 2k - \frac{5}{2} \cdot \frac{1}{k}$$

$$\frac{1}{y} \cdot \frac{dy}{dk} = 1 + \frac{2 \cos 2k}{\sin 2k} - \frac{5}{2k}$$

$$\frac{dy}{dk} = y \left[1 + \frac{2 \cos 2k}{\sin 2k} - \frac{5}{2k} \right]$$

$$\therefore \frac{dy}{dk} = \left[\frac{3e^k \sin 2k}{k^{5/2}} \right] \left[1 + 2 \cot 2k - \frac{5}{2k} \right]$$

Integrate the following with respect to the Variable

Question 1

$$4 \sec^2(3m+1)$$
$$\int 4 \sec^2(3m+1) dm$$
$$4 \int \sec^2(3m+1) dm$$

$$u = 3m + 1$$

$$\frac{du}{dm} = 3$$

$$dm$$

$$dm = \frac{du}{3}$$

$$4 \int \sec^2 u \cdot \frac{du}{3}$$

$$\frac{4}{3} \int \sec^2 u du$$

$$\frac{4}{3} [\tan u] + C$$

$$\therefore \int 4 \sec^2(3m+1) dm = \frac{4}{3} \tan(3m+1) + C$$

Question 2

$$2t(3t^2-1)^{1/2}$$

$$\int 2t(3t^2-1)^{1/2} dt$$

$$2 \int t(3t^2-1)^{1/2} dt$$

$$u = (3t^2-1)^{1/2}$$

$$u^2 = 3t^2 - 1$$

$$u^2 + 1 = 3t^2$$

$$t^2 = \frac{u^2 + 1}{3}$$

$$t = \left(\frac{u^2 + 1}{3} \right)^{1/2}$$

$$\frac{dt}{du} = \frac{1}{2} \left(\frac{u^2 + 1}{3} \right)^{-1/2} \cdot \frac{2u}{3}$$

$$\frac{dt}{du} = \frac{u}{3} \left(\frac{u^2+1}{3} \right)^{-1/2}$$

$$dt = \frac{u du}{3} \left(\frac{u^2+1}{3} \right)^{-1/2}$$

$$2 \int \left(\frac{u^2+1}{3} \right)^{1/2} \cdot \frac{u}{3} \cdot u du \left(\frac{u^2+1}{3} \right)^{-1/2}$$

$$\frac{2}{3} \int \left(\frac{u^2+1}{3} \right)^{1/2-1/2} \cdot u^2 du$$

$$\frac{2}{3} \int u^2 du$$

$$\frac{2}{3} \left[\frac{u^3}{3} \right] + C$$

$$\therefore \int 2t(3t^2-1)^{1/2} dt = \frac{2(3t^2-1)^{3/2}}{3} + C$$

Question 3

$$2 \int \frac{2x}{\sqrt{4x^2-1}} dx$$

$$u = \sqrt{4x^2-1}$$

$$u^2 + 1 = 4x^2$$

$$x = \left(\frac{u^2+1}{4} \right)^{1/2}$$

$$\frac{dx}{du} = \frac{1}{2} \left(\frac{u^2+1}{4} \right)^{-1/2} \cdot \frac{u}{2}$$

$$dx = \frac{u du}{4} \left(\frac{u^2+1}{4} \right)^{-1/2}$$

$$2 \int \left(\frac{u^2+1}{4} \right)^{1/2} \cdot \frac{1}{u} \cdot \frac{u du}{4} \left(\frac{u^2+1}{4} \right)^{-1/2}$$

$$= \frac{2}{4} \int \left(\frac{u^2+1}{4} \right)^{1/2-1/2} \cdot \frac{1}{u} u du$$

$$\frac{1}{2} \int du$$

$$= \frac{1}{2} [u] + C$$

$$\therefore \int \frac{2x}{(4x^2-1)^{1/2}} dx = \frac{\sqrt{4x^2-1}}{2} + C$$

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