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1 Find the limit of the function (as x tends to zero, as x → 0)

Solution

$$\lim_{x \to 0} \frac{(x - \cos x)}{x}$$

$$\lim_{x \to 0} \frac{(1 - \cos x)}{1}$$

$$= \lim_{x \to 0} \frac{(1 + \sin x)}{1} = \frac{1 + \sin(0)}{1} = \frac{1}{1} = 1$$

2 If $y = 3\tan 7x e^{3x}$, Find $dy/dx$

Solution

$y = 3 \quad U = \tan 7x \quad v = e^{3x}$

$d/dx = 0 \quad du/dx = 7\sec^2 7x \quad dv/dx = 3e^{3x}$

$$\frac{dy}{dx} = d\left[\frac{1}{3}(v) + \frac{1}{\tan 7x}(7\sec^2 7x) + \frac{1}{e^{3x}}(3e^{3x})\right]$$

$$\frac{dy}{dx} = [y]\left[0 + \frac{7\sec^2 7x}{\tan 7x} + 3\right]$$

$$\frac{dy}{dx} = \left[\frac{7\sec^2 7x}{\tan 7x} + 3\right]\left[-3\tan 7x e^{3x}\right]$$

3 If $y = \cos 3x$. Find $dy/dx$ from first principle

Solution

$y = \cos 3x$

$y + \delta y = \cos(3x + \delta x)$

$\delta y = \cos(3x + \delta x) - \cos 3x$ — (i)

Recall $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

4 $f(x) = x^2 - 1$ and $g(x) = -3x$

$(f-g)(5) = f(5) - g(5)$

$(f-g)(5) = [5^2 - 1(5)] - [-3(5)]$

$(f-g)(5) = [25 - 5] - [-15]$

$(f-g)(5) = 20 + 15$

$(f-g)(5) = 35$

5 Find $f(g(x))$ if $f(x) = 4x^2 + 2$ and $g(x) = 3x + 3$

Solution

$f(g(x)) = 4[3x + 3]^2 + 2$

$= 4[9x^2 + 18x + 9] + 2$

$= 36x^2 + 72x + 36 + 2$

$= 36x^2 + 72x + 38$

6 Find $\frac{dy}{dx}$ of $x^2 + 2xy + y^2 = 100$

$2x^{2-1} + 2x^{1-1}y + 2y\frac{dy}{dx} + 2y^{1-1}\frac{dy}{dx} = 0$

$2x + 1x^0 y + 2y + 2y^0\frac{dy}{dx} = 0$

$\left(2x\frac{dy}{dx}\right) + \left(2y\frac{dy}{dx}\right) = -2x - 2y$

$\frac{dy}{dx}(2x + 2y) = -2x - 2y$

$\frac{dy}{dx} = \frac{-2x - 2y}{2x + 2y}$

7 Find first derivative of the function of $y = x^2 \cos x$

$y = x^2 \cos x$

1st derivative $= 2x - \sin x$

Comparing equation 1 & 2

$A = 3x + \Delta x$ $\qquad B = 3x$

$\frac{A+B}{2} = \frac{3x + \Delta x + 3x}{2} \qquad \frac{6x + \Delta x}{2} = 6x + \frac{\Delta x}{2}$

and

$\frac{A-B}{2} = \frac{3x + \Delta x - 3x}{2} = \frac{\Delta x}{2}$

Hence

$\Delta y = \cos(3x + \Delta x) - \cos 3x$

$= -2 \sin\left(6x + \frac{\Delta x}{2}\right) \sin \frac{\Delta x}{2}$

Divide through by $\Delta x$ $\quad \frac{\Delta y}{\Delta x} = \left[-2 \sin\left(6x + \frac{\Delta x}{2}\right)\right]\left(\sin \frac{\Delta x}{2}\right]$

$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1, \quad \frac{\sin B}{B} = 1$

$$\frac{\Delta y}{\Delta x} = \frac{-2 \sin\left(6x + \frac{\Delta x}{2}\right)\left(\sin \frac{\Delta x}{2}\right) \times \frac{1}{2}}{\Delta x \times \frac{1}{2}}$$

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = -2 \sin(6x + 0) \lim_{\Delta x \to 0} \sin\left(\frac{\Delta x}{2}\right) / \Delta x/2$

Since $\lim_{\Delta x \to 0} \sin\left(\frac{\Delta x}{2}\right) = 1$

$\therefore dy/dx = -2 \sin 6x$

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