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Mat 104

1) $\lim_{x \to 0} \left\{ \frac{x - \cos x}{x} \right\}$

Soln

$\lim_{x \to 0} \left\{ \frac{x - \cos x}{x} \right\}$

Using direct substitution, we have undefined but

Using Lhopital's rule

$\lim_{x \to 0} \left\{ \frac{x - \cos x}{x} \right\} = \lim_{x \to 0} \left\{ \frac{1 - (-\sin x)}{1} \right\}$

$= \lim_{x \to 0} \left\{ \frac{1 + \sin x}{1} \right\} = \frac{1 + \sin 0}{1} = \frac{1+0}{1} = \underline{\underline{1}}$

2) $y = -3\tan 7x e^{3x}$

Solu

$y = -3\tan 7x\, e^{3x}$

Let $u = -3$, $v = \tan 7x$, $w = e^{3x}$

$\frac{du}{dx} = 0$, $\frac{dv}{dx} = 7\sec^2 7x$, $\frac{dw}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = y\left\{ \frac{1}{u} \times \frac{du}{dx} + \frac{1}{v} \frac{dv}{dx} + \frac{1}{w} * \frac{dw}{dx} \right\}$$

$$dy/dx = y\left\{ \frac{1}{-3} \times 0 + \frac{1}{\tan 7x} \times 7\sec^2 x\, 7x + \frac{1}{e^{3x}} \times 3e^{3x} \right.$$

$$dy/dx = y\left[0 + \frac{7\sec^2 7x}{\tan 7x} + 3 \right]$$

$$\frac{dy}{dx} = -3\tan 7x\, e^{3x} \left\{ \frac{7\sec^2 7x}{\tan 7x} + 3 \right\}$$

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3) $y = \cos 3x$ from the first principle

Sdu

$y = \cos 3x$

$y + \Delta y = \cos 3(x + \Delta x)$

$y + \Delta y = \cos 3x + 3\Delta x$

$\Delta y = \cos 3x + 3\Delta x - \cos 3x$ --- (1)

Recall $\cos - \cos B = 2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$ --- (2)

Compare equ (1) and (2)

$A = 3x + 3\Delta x$, $B = 3x$

$\frac{A+B}{2} = \frac{3x + 3\Delta x + 3x}{2}$ $= \frac{6x + 3\Delta x}{2} = \frac{3x + 3\Delta x}{2}$

$= 3x + \frac{3\Delta x}{2}$

$\frac{A-B}{2} = \frac{3x + 3\Delta x - 3x}{2} = \frac{3\Delta x}{2}$

Hence $\Delta y = -2 \sin \left(3x + \frac{3\Delta x}{2}\right) \sin \left(\frac{3\Delta x}{2}\right)$

divide through by Δx --- *

$\Delta y / \Delta x = -2 \sin \left(3x + \frac{3\Delta x}{2}\right) \sin \left(\frac{3\Delta x}{2}\right)$

Δx

Multiply both numerator and denominator through by $3/2$ --- *

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$$-2\sin\left(3x+\frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)\times 3/2$$

$$\Delta x \times 3/2$$

$$\Delta y/\Delta x = -3\sin\left(3x+\frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)$$

$$\frac{3\Delta x}{2}$$

$$\lim_{\Delta x \to 0} \Delta y/\Delta x = -3\sin 3x$$

$$\lim_{\to 0} \frac{\sin \frac{3\Delta x}{2}}{\frac{3\Delta x}{2}} = 1$$

Hence $dy/dx = -3\sin 3x$

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4) $f(x) = 2x^3 - 7x$

$g(x) = -3x$

$f(x) - g(x) = 2x^3 - 7x - (-3x)$

$= 2x^3 - 7x + 3x$

$= 2x^3 - 4x$

$(f-g)(5) = (2x^3 - 4x) \times 5$

$= 10x^3 - 20x$

(d.t.b) divide through by 10

$x^3 - 2x$

5) $f \circ g(x)$

$f(x) = 4x^2 + 2$

$g(x) = 2x + 3$

$f \circ g(x) = 4(2x+3)^2 + 2$

$= 4(4x^2 + 12x + 9) + 2$

$= 16x^2 + 48x + 36 + 2$

$= 16x^2 + 48x + 38$

$fog(x) = 16x^2 + 48x + 38$

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6) $x^2 + 2xy + y^2 = 1020$

Solu

$x^2 + 2xy + y^2 = 1020$

$2x\frac{dx}{dx} + 2x\frac{dy}{dx} + 2y\frac{dx}{dx} \quad \frac{d}{dx} = 0$

$2x + 2x\,dy/dx + 2y\,dy/dx = 0$

$2x + 2y + dy/dx(2x + 2y) = 0$

$dy/dx(2x + 2y) = -2x - 2y$

$dy/dx = \underline{-2x - 2y}$

$\underline{2x + 2y}$

7) $y = x^2 \cos x$

Solu

Put $u = x^2$, $V = \cos x$

$du/dx = 2x$, $dv/dx = -\sin x$

Recall $u \cdot dv/dx + V \cdot du/dx$

$= x^2 \cdot -\sin x + \cos x \cdot 2x$

$= -x^2 \sin x + 2x \cos x$

$= 2x\cos x - x^2 \sin x$

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