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1) $y = \frac{(x+1)^2(x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}$

$\ln y = \ln(x+1)^2 + \ln(x-2)^{1/2} - \ln(2x-1) - \ln(x-3)^{4/3}$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{2(x+1)^1}{(x+1)^2} + \frac{(x-2)^{-1/2}}{2(x-2)^{1/2}} - \frac{2}{2x-1} - \frac{4(x-3)^{1/3}}{3(x-3)^{4/3}}$

$\frac{dy}{dx} = y\left[\left(\frac{2}{x+1}\right) + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{1}{3(x-3)}\right]$

$\Rightarrow \frac{dy}{dx} = \frac{(x+1)^2(x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}\left[\frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{1}{3(x-3)}\right]$

2) $y = \frac{[3e^k \sin 2k]}{k^{5/2}}$

$\ln y = \ln 3e^k + \ln \sin 2k - \ln k^{5/2}$.

$\frac{1}{y}\cdot\frac{dy}{dk} = \frac{3e^k}{3e^k} + \frac{2\cos 2k}{\sin 2k} - \frac{5k^{3/2}}{2k^{5/2}}$.

$\frac{dy}{dk} = y\left(1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2k}\right)$.

$\therefore \frac{dy}{dk} = \frac{3e^k \sin 2k}{k^{5/2}}\left(1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2k}\right)$.

Integrate the following with respect to the variable

1.) $4\sec^2(3m+1)$

$\Rightarrow \int 4\sec^2(3m+1)\, dm$.

let $U = 3m + 1$

$\frac{du}{dm} = 3 \quad \therefore dm = \frac{du}{3}$.

$\int 4\sec^2(U) \frac{du}{3} \Rightarrow \frac{4}{3}\int \sec^2(U)\, du$.

remember, integral of $\sec^2 U = \tan U + C$.

$\Rightarrow \frac{4}{3}(\tan U) + C$

$\Rightarrow \frac{4}{3}\tan U + C$ remember $U = 3m + 1$.

$\therefore$ integral $= \frac{4}{3}\tan(3m+1) + C$.

2.) $U = 3t^2 - 1$

$\frac{du}{dt} = 6t$

$\therefore dt = \frac{du}{6t}$.

Question :- $2t(3t^2-1)^{1/2}$.

$\int 2t(3t^2-1)^{1/2}\, dt$

$\Rightarrow \int \cancel{2t}(U)^{1/2} \frac{du}{\cancel{6t}_3} \Rightarrow \int U^{1/2} \frac{du}{3}$

$\Rightarrow \frac{1}{3}\left(\frac{U^{1/2+1}}{3/2}\right) + C \Rightarrow \frac{2U^{3/2}}{9}$. remember $U = 3t^2 - 1$

$\therefore \frac{2(3t^2-1)^{3/2}}{9}$.

3.) $\frac{2x}{(4x^2-1)^{1/2}} \Rightarrow \int 2x(4x^2-1)^{-1/2}\,dx$

Let $u = 4x^2 - 1$

$du/dx = 8x$

$dx = du/8x$

$\Rightarrow \int \cancel{2x}\,(u)^{-1/2}\,\frac{du}{\cancel{8x}_4} \Rightarrow \frac{1}{4}\int u^{-1/2}\,du.$

$\Rightarrow \frac{1}{4}\left(\frac{u^{-1/2+1}}{-1/2+1}\right) + C. \quad \frac{1}{4}\left(\frac{u^{1/2}}{1/2}\right)$

$\Rightarrow \frac{2u^{1/2}}{4} \Rightarrow \frac{u^{1/2}}{2} + C$

remember $u = 4x^2 - 1$.

$\Rightarrow \frac{(4x^2-1)^{1/2}}{2} + C.$