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19/ENG05/066

1 $\lim_{x \to 0} \left\{ \frac{x - \cos x}{x} \right\}$

Solution

$\lim_{x \to 0} \left\{ \frac{x - \cos x}{x} \right\}$

By direct substitution, we have $\frac{0}{0}$: Using Lhopital's rule

$\lim_{x \to} \left\{ \frac{x - \cos x}{x} \right\} = \lim_{x \to 0} \left\{ \frac{1 - (-\sin x)}{1} \right\}$

$= \lim_{x \to 0} \left\{ \frac{1 + \sin x}{1} \right\} = \frac{1 + \sin 0}{1} = \frac{1 + 0}{1}$

$= 1$

2 $y = -3 \tan 7x e^{3x}$

Solution

$y = -3 \tan 7x e^{3x}$

$U = -3$ $\quad V = \tan 7x$ $\quad w = e^{3x}$

$\frac{du}{dx} = 0$ $\quad \frac{dV}{dx} = 7 \sec^2 7x$ $\quad \frac{dw}{dx} = 3e^{3x}$

$\frac{dy}{dx} = y \left[\frac{1}{u} \times \frac{dy}{dx} + \frac{1}{v} \times \frac{dy}{dx} + \frac{1}{w} \times \frac{dw}{dx} \right]$

$= y \left[\frac{1}{-3} \times 0 + \frac{1}{\tan 7x} \times 7 \sec^2 7x + \frac{1}{e^{3x}} \times 3e^{3x} \right.$

$= y \left[0 + \frac{7 \sec^2 7x}{\tan 7x} + 3 \right]$

$\frac{dy}{dx} = -3 \tan 7x e^{3x} \left[\frac{7 \sec^2 7x}{\tan 7x} + 3 \right]$

7 Continued

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3 $y = \cos 3x$

Solution

$y = \cos x$

$y + \Delta y = \cos 3(x + \Delta x)$

$y + \Delta y = \cos x + 3\Delta x$

$\Delta y = \cos x + 3\Delta x - \cos 3x$ ----- ①

Recall $\cos A - \cos B = 2\sin \frac{A+B}{2} \sin \frac{A-B}{2}$ ---- ②

Comparing ① and ②

$A = 3x + 3\Delta x$ $B = 3x$

$\frac{A+B}{2} = \frac{3x + 3\Delta x + 3x}{2} = \frac{6x + 3\Delta x}{2} = 3x + \frac{3\Delta x}{2}$

$\frac{A-B}{2} = \frac{3x + 3\Delta x - 3x}{2} = \frac{3\Delta x}{2}$

Hence $\Delta y = -2\sin\left(3x + \frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)$

divide through by Δx

$$\frac{\Delta y}{\Delta x} = \frac{-2\sin\left(3x + \frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)}{\Delta x}$$

multiply through by $\frac{3}{2}$ both numerators and denominator

$$\frac{-2\sin\left(3x + \frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)\frac{3}{2}}{\Delta x \times \frac{3}{2}}$$

3 Continued

$$\frac{\Delta y}{\Delta x} = \frac{-3\sin\left(3x + \frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)}{\frac{3\Delta x}{2}}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = -3\sin 3x$$

$$\lim_{x \to 0} \frac{\sin \frac{3\Delta x}{2}}{\frac{3\Delta x}{2}} = 1$$

$\therefore -3\sin 3x \times 1$

Hence $\frac{dy}{dx} = -3\sin 3x$

4 $f(x) = 2x^3 - 7x$

$g(x) = -3x$

$$f(x) - g(x) = 2x^3 - 7x - (-3x)$$
$$= 2x^3 - 7x + 3x$$
$$= 2x^3 - 4x$$

$$(f-g)(5) = (2x^3 - 4x) \times 5$$
$$= 10x^3 - 20x$$

dividing through by 10

$x^3 - 2x$

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5 $Fog(x)$

$F(x) = 4x^2+2$

$g(x) = 2x+3$

$$Fog(x) = 4(2x+3)^2+2$$
$$= 4(4x^2+12x+9)+2$$
$$= 16x^2+48x+36+2$$
$$= 16x^2+48x+38$$

$$Fog(x) = 16x^2+48x+38$$

6 $x^2+2xy+y^2 = 1020$

Solution

$$x^2+2xy+y^2 = 1020$$

$$2x\frac{dx}{dx} + 2\frac{dy}{dx} + 2y\frac{dx}{dx} + 2y\frac{dy}{dx} = 0$$

$$2x + 2x\,dy/dx + 2y + 2y\frac{dy}{dx} = 0$$

$$2x+2y+\frac{dy}{dx}(2x+2y = 0$$

$$\frac{dy}{dx}(2x+2y) = -2x-2y$$

$$\frac{dy}{dx} = \frac{-2x-2y}{2x+2y}$$

5 Fog(x)

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7 $y = x^2 \cos x$

Solution

Put $u = x^2$, $v = \cos x$

$\frac{du}{dx} = 2x$, $\frac{dv}{dx} = -\sin x$

6

Recall $u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$

$= x^2 \cdot -\sin x + \cos x \cdot 2x$

$= -x^2 \sin x + 2x \cos x$

$= 2x \cos x - x^2 \sin x$