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Course Code: Math 104

1. Find the limit of $(x - \cos x)/x$ as $x \rightarrow 0$

Solution:

$$\lim_{x \rightarrow 0} \frac{x - \cos x}{x}$$

$$\lim_{x \rightarrow 0} \frac{x - \cos x}{x} \times \frac{1 + \cos x}{1 + \cos x}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x(1 + \cos x)}$$

Since $1 - \cos^2 x = \sin^2 x$

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{x(1 + \cos x)}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{\sin x}{1 + \cos x}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} \quad \lim_{x \rightarrow 0} \frac{\sin x}{1 + \cos x}$$

$$\frac{\sin(0)}{0} \cdot \frac{\sin(0)}{1 + \cos(0)}$$

$$\frac{0}{0} \cdot \frac{0}{1+1} = 0$$

$$\therefore \text{Ans} = \underline{\underline{0}}$$

2. Find $\frac{dy}{dx}$ of $-3t + 7x + 3x$

3. $\cos 3x$ First principle.

$$f(x) = \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \frac{\cos 3x(x+h) - \cos 3x}{h}$$

$$= \frac{\cos 3x \cos 3h - \sin 3x \sin 3h - \cos 3x}{h}$$

$$\frac{\cos 3x (\cos 3h - 1)}{h} - \lim_{h \rightarrow 0} \sin 3x \sin h$$

$$\cos 3x \lim_{h \rightarrow 0} (\cos 3h - 1)/h - \sin 3x \cdot \lim_{h \rightarrow 0} \frac{\sin 3h}{h}$$
$$= \cos 3x \times 0 - \sin 3x \times 1$$

Since $\lim_{h \rightarrow 0} \sin 3h/h = 1$ and $\lim_{h \rightarrow 0} (\cos 3h - 1)/3h = 0$

$$= 0 - \sin 3x$$

4. $F(x) = 2x^3 - 7x$, $g(x) = 2x + 3$. Find $(f-g)(5)$

$$(f-g)$$
$$(2x^3 - 7x) - (2x + 3)$$

$$= 2x^3 - 7x - 2x - 3$$

$$= 2x^3 - 9x - 3$$

$$x = 5$$

$$2(5)^3 - 9(5) - 3$$

$$250 - 45 - 3$$

$$= 202$$

5. $F(x) = 4x^2 + 2$, $g(x) = 2x + 3$. Find $f \circ g$.

Solution

$$F(x) = 4x^2 + 2 \quad g(x) = 2x + 3$$

$$F(g(x)) = F(2x + 3)$$

$$= 4x^2 (2x + 3) + 2$$

$$= 8x^3 + 12x^2 + 2.$$

6. $x^2 + 2xy + y^2 = 1,020$

$$x^2 + y^2 + 2xy = 1,020.$$

7. Find the first derivative of the function $y = x^2 \cos x$

Solution

Using product rule.

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = x^2 \quad \frac{du}{dx} = 2x \quad u = \cos x \quad \frac{dv}{dx} = -\sin x$$

$$\frac{dy}{dx} = x^2 (-\sin x) + (\cos x) (2x)$$

$$\frac{dy}{dx} = -x^2 \sin x + 2x \cos x$$

$$\frac{dy}{dx} = 2x \cos x - x^2 \sin x$$