Odunlami Olaniyi Obadaya

Mechanical Engineering 19/ENG06/040

MAT 104:

1.) Differentiate $y = \frac{[(x+1)^2(x-2)^{1/2}]}{[(2x-1)(x-3)^{4/3}]}$

$\ln y = \ln(x+1)^2 + \ln(x-2)^{1/2} - \ln(2x-1) - \ln(x-3)^{4/3}$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{(x+1)^2} \cdot 2(x+1) + \frac{1}{\sqrt{x-2}} \cdot \frac{1}{2}(x-2)^{-1/2} - \frac{1}{2x-1} \cdot 2 - \frac{1}{(x-3)^{4/3}} \cdot \frac{4}{3}(x-3)^{1/3}$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{(x+1)} + \frac{1}{2(x-2)^{1/2}(x-2)^{1/2}} - \frac{2}{2x-1} - \frac{4}{3(x-3)}$

$\frac{dy}{dx} = y\left[\frac{2}{(x+1)} + \frac{1}{2(x-2)^{1/2}(x-2)^{1/2}} - \frac{2}{2x-1} - \frac{4}{3(x-3)}\right]$

$\frac{dy}{dx} = \frac{[(x+1)^2(x-2)^{1/2}]}{[(2x-1)(x-3)^{1/2}]}\left[\frac{2}{(x+1)} + \frac{1}{2(x-2)^{1/2}(x-2)^{1/2}} - \frac{2}{2x-1} - \frac{4}{3(x-3)}\right]$

2.) $y = \frac{[3e^k \sin 2k]}{k^{5/2}}$

$\ln y = \ln(3e^k) + \ln(\sin 2k) - \ln(k^{5/2})$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{3e^k} \cdot 3e^k + \frac{1}{\sin 2k} \cdot 2\cos 2k - \frac{1}{k^{5/2}} \cdot \frac{5}{2}k^{3/2}$

$\frac{1}{y}\frac{dy}{dx} = 1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2k}$

$\frac{dy}{dx} = y\left[1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2k}\right]$

$\frac{dy}{dx} = \frac{[3e^k \sin 2k]}{k^{5/2}}\left[1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2k}\right]$

3.) ~~Integrate~~ $\int 4\sec^2(3m+1)\,dm$

$4\int \sec^2(3m+1)\,dm$

Let $u = 3m+1$

$\frac{du}{dm} = 3 \quad \therefore dm = \frac{du}{3}$

4. $\int \sec^2 u \frac{du}{3}$

$\frac{4}{3}\int \sec^2 u \, du$

$\frac{4}{3}\tan u + c$

$= \frac{4}{3}\tan(3m+1) + c$

4. $\int 2t(3t^2-1)^{1/2} \, dt$

Let $u = (3t^2-1)$

$\frac{du}{dt} = 6t$

$dt = \frac{du}{6t}$

$\int 2t(u)^{1/2} \frac{du}{6t}$

$\frac{1}{3}\int u^{1/2} \, du = \frac{1}{3}\frac{u^{1/2+1}}{1/2+1} + c$

$= \frac{1}{3}\frac{u^{3/2}}{3/2} + c$

$= \frac{1}{3}\frac{2u^{3/2}}{3} + c = \frac{2u^{3/2}}{9} + c$

$= \frac{2(3t^2-1)^{3/2}}{9} + c$

5. ~~$\int 2x$~~ $\int \frac{2x}{(4x^2-1)^{1/2}} \, dx$

$u = 4x^2 - 1$

$\frac{du}{dx} = 8x$

$dx = \frac{du}{8x}$

$\int \frac{2x}{u^{1/2}} \frac{du}{8x}$

$\frac{1}{4}\int \frac{1}{u^{1/2}} \, du$

$$\frac{1}{4}\int u^{-1/2}\,du$$

$$= \frac{1}{4}\,\frac{u^{1/2}}{1/2} + c$$

$$= \frac{2}{4}\cdot u^{1/2} + c$$

$$= \frac{u^{1/2}}{2} + c$$

$$= \frac{(4x^2-1)^{1/2}}{2} + c$$