COURSE: MAT 104
COURSE LECTURER: DR OYELAMI
DATE SUBMITTED: 3rd April, 2020
NAME: ELUAGU GERALD TOBENNA
DEPARTMENT: MECHATRONICS ENGINEERING
MATRIC NO: 19/ENG05/026

1. Find the limit of the function $(x-\cos x)/x^3$ as $x \to 0$
2. If $y = -3\tan 3x$ find dy/dx
3. If $y = \cos 3x$, find dy/dx from the first principle
4) Given that $f(x) = 2x^3 - 7x$ and $g(x) = -3x$ find $(f-g)(5)$
5) Find $fog(x)$ if $f(x) = 4x^2 + 2$ and $g(x) = x+3$
6) Find the gradient of $x^2 + 2xy + y^2 = 1020$
7) Find the first derivative of the function $y = x^2 \cos x$

P.T.O.

3) $y = \cos 3x$

From, first principle,

$y + \Delta y = \cos 3(x + \Delta x)$

$\Delta y = \cos 3(x + \Delta x) - y$

$\Delta y = \cos(3x + \Delta x) - \cos 3x \quad -(i)$

$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \quad -(ii)$

$\Delta y = -2\sin\left(\frac{3x + 3\Delta x + 3x}{2}\right)\sin\left(\frac{3x + 3\Delta x - 3x}{2}\right)$

$\Delta y = -2\sin\left(\frac{6x + 3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)$

Dividing through by Δx

$$\frac{\Delta y}{\Delta x} = \frac{-2\sin\left(\frac{6x + 3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)}{\Delta x}$$

Multiply both numerator and denominator by $1/2$

$$\frac{\Delta y}{\Delta x} = \frac{-2\sin\left(\frac{6x + 3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right) \times \frac{1}{2}}{\Delta x \times 1/2}$$

$$\frac{\Delta y}{\Delta x} = -\sin(3x + 3\Delta x/2)\,\frac{\sin(3\Delta x/2)}{\Delta x/2}$$

$$\frac{\Delta y}{\Delta x} = -\sin(3x + 3\Delta x/2)\,\frac{\sin(3\Delta x/2)}{\Delta x/2}$$

$$= -\sin(3x + 3\Delta x/2)\,\frac{\sin 3(\Delta x/2)}{\Delta x/2}$$

1. By direct substitution

$$\lim_{x\to 0}\left\{\frac{x-\cos x}{x}\right\} = \frac{0-\cos 0}{0} = \frac{0-1}{0} = \frac{-1}{0}$$ undefined

since it is undefined

using L'Hopital's Rule

$$\lim_{x\to 0}\left\{\frac{x-\cos x}{x}\right\} = \lim_{x\to 0}\left\{\frac{1-(-\sin x)}{1}\right\}$$

$$= \lim_{x\to 0}\left\{\frac{1+\sin x}{1}\right\} = \frac{1+\sin(0)}{1} = \frac{1+0}{1} = \frac{1}{1}$$

$$= 1$$

2. $y = -3\tan 7x e^{3x}$, find dy/dx

$$y = -3\left[\tan 7x e^{3x}\right]$$

$u = -3$, $v = \tan 7x$ $\quad w = e^{3x}$

$\frac{du}{dx} = 0$, $\frac{dv}{dx} = 7\sec^2 7x$, $\frac{dw}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = y\left[\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} + \frac{1}{w}\frac{dw}{dx}\right]$$

$$= y\left[\frac{1}{-3}(0) + \frac{1}{\tan 7x}(7\sec^2 7x) + \frac{1}{e^{3x}}(3e^{3x})\right]$$

$$= y\left[0 + \frac{7\sec^2 7x}{\tan 7x} + 3\right] = -3\tan 7x e^{3x}\left(\frac{\sec^2 7x}{\tan 7x} + 3\right)$$

$$\frac{dy}{dx} = -21 e^{3x}\sec^2 7x - 9\tan 7x e^{3x}$$

Eluagu Gerald Tobenna
Mechatronics Engineering

6) $x^2 + 2xy + y^2 = 1020$

$2x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 0$

$\therefore 2x + 2y + \frac{dy}{dx}(2x+2y) = 0$

$\frac{dy}{dx}(2x+2y) = 0 - 2x - 2y$

$\frac{dy}{dx} = \frac{0 - 2x - 2y}{2x+2y}$

$\frac{dy}{dx} = \frac{2(0 - x - y)}{2(x+y)}$

$\frac{dy}{dx} = \frac{-(x+y)}{(x+y)}$

$\frac{dy}{dx} = -1$

7) $y = x^2\cos x$

let $u = x^2$, $v = \cos x$ $du/dx = 2x$ $dv/dx =$

product rule $= u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{dy}{dx} = x^2(-\sin x) + \cos x(2x)$

$dy/dx = -x^2\sin x + 2x\cos x$

Taking limits as $\Delta x \to 0$, we have

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = -\sin 3x \lim_{\Delta x \to 0} \frac{\sin 3(\Delta x/2)}{\Delta x/2}$$

$$\lim_{\Delta x \to 0} \frac{\sin 3(\Delta x/2)}{\Delta x/2} = 3$$

Hence

$$\frac{dy}{dx} = -\sin 3x \,(3)$$

$$= \underline{-3\sin 3x}$$

4) $f(x) = 2x^3 - 7x \quad g(x) = -3x$

$(f-g)(5) = f(5) - g(5)$

$= 2(5)^3 - 7(5) - -3(5)$

$= 2(125) - 35 - (-15)$

$250 - 35 - (-15)$

$215 + 15 = \underline{230}$

5) Find $fog(x)$ if $f(x) = 4x^2 + 2$ and $g(x) = 2x + 3$

$g(x) = 2x + 3 \quad f(x) = 4x^2 + 2$

$fog(x) = 4(2x+3)^2 + 2 = 4(2x+3)(2x+3) + 2$

$= 4(4x^2 + 6x + 6x + 9) + 2 = 4(4x^2 + 12x + 9) + 2$

$= 16x^2 + 48x + 36 + 2$

$= \underline{16x^2 + 48x + 38}$