

1) Find the limit of the functions

Solution

$$\lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \cos x)}{x(1 + \cos x)} = \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x(1 + \cos x)}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x(1 + \cos x)} = \lim_{x \rightarrow 0} \frac{\sin^2 x (\sin x)(\cos x)}{x(1 + \cos x)}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{\sin x}{1 + \cos x}$$

$$= 1 \cdot 1 = 1$$

2) If $y = -3 \tan^{-1} x e^{3x}$, find $\frac{dy}{dx}$

Solution

Applying product rule $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

$$\frac{dy}{dx} = 3e^{3x} (\tan^{-1} x) + e^{3x} (7 \sec^2 x)$$

$$= 3e^{3x} \tan^{-1} x + 7e^{3x} \sec^2 x$$

If $y = \cos 3x$, find $\frac{dy}{dx}$ from the first principle

Solution

$$y = \cos 3x \quad \cos u$$

$$u = 3x, \quad du = 3$$

$$y' = du - 5 \sin x$$

$$y' = -3 \sin 3x$$

4) Given that $f(x) = 2x^3 - 7x$ and $g(x) = -3x$, find $(f-g)(s)$

Solution

$$f(x) = 2x^3 - 7x$$

$$g(x) = -3x$$

$$(f-g)(s) = f(s) - g(s)$$

$$= (2s^3 - 7s) - (-3s)$$

$$= 2s^3 - 6s - 15s$$

5) Find $f \circ g(x)$ if $f(x) = 4x^2 + 2$ and $g(x) = 2x + 3$

Solution

$$f \circ g(x) = f(g(x))$$

$$= 4(2x+3)^2 + 2$$

$$= 4(4x^2 + 16 + 12) + 2$$

$$= 16x^2 + 64x + 48 + 2$$

$$= 16x^2 + 64x + 50$$

6) Find the gradient of $x^2 + 2xy + y^2 = 10z$

Solution

$$\frac{d}{dx} (x^2 + 2xy + y^2) = \frac{d}{dx} (10z)$$

$$\frac{d}{dx} x^2 + \frac{d}{dx} 2xy + \frac{d}{dx} y^2 = 0$$

$$A \rightarrow 2x + 2y + 0 = 0$$

$$B \rightarrow 0 + 2x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$\text{Add A + B}$$

$$2x + 2y + 2x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = -1$$