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MAT 104
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1) $y = \frac{(x+1)^2 (x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}$

$\ln y = [\ln|(x+1)^2| + \ln(x-2)^{1/2}] - [\ln(2x-1) + \ln(x-3)^{4/3}]$

$\frac{1}{y}\frac{dy}{dx} = \left[\frac{1}{(x+1)^2} \cdot 2(x+1) + \frac{1}{(x-2)^{1/2}} \cdot \frac{(x-2)^{-1/2}}{2}\right] - \left[\frac{1}{(2x-1)} \cdot 2 + \frac{1}{(x-3)^{4/3}} \cdot 4(x-3)^{1/3}\right]$

$\frac{1}{y}\frac{dy}{dx} = \left[\frac{2(x+1)}{(x+1)^2} + \frac{\frac{1}{\sqrt{x-2}}}{2\sqrt{x-2}}\right] - \left[\frac{2}{2x-1} + \frac{4(x-3)^{1/3}}{3(x-3)^{4/3}}\right]$

$\frac{1}{y}\frac{dy}{dx} = \left[\frac{2}{(x+1)} + \frac{1}{2(x-2)}\right] - \left[\frac{2}{(2x-1)} + \frac{4}{3(x-3)}\right]$

$\frac{dy}{dx} = y\left[\frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{4}{3(x-3)}\right]$

$\frac{dy}{dx} = \frac{(x+1)^2 (x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}\left[\frac{2}{(x+1)} + \frac{1}{(2x-4)} - \frac{2}{(2x-1)} - \frac{4}{(3x-9)}\right]$

2) $y = \frac{3e^k \sin 2k}{x^{5/2}}$

$\ln y = \ln(3e^k) + \ln(\sin 2k) - \ln(x^{5/2})$

$\frac{1}{y}\cdot\frac{dy}{dk} = \frac{1}{3e^k} 3e^k + \frac{1}{\sin 2k} \cdot 2\cos 2k - \frac{1}{k^{5/2}} \cdot \frac{5}{2} k^{3/2}$

$\frac{1}{y}\cdot\frac{dy}{dk} = 1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2} k^{\frac{3}{2}-\frac{5}{2}}$

$\frac{dy}{dk} = y\left[1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2} k^{-1}\right]$

$$\frac{dy}{dK} = \frac{3e^K \sin 2K}{K^{5/2}} \left[1 + \frac{2\cos 2K}{\sin 2K} - \frac{5}{2K} \right]$$

3 $\int 4\sec^2(3m+1)\,dm$

$4\int \sec^2(3m+1)\,dm$

$u = 3m + 1$

$\frac{du}{dm} = 3$

$du = 3\,dm$

$dm = \frac{du}{3}$

$4\int \sec^2(u)\frac{du}{3}$

$4/3 \int \sec^2(u)\,du$

$= \frac{4}{3}\tan u + C$

$= \frac{4}{3}\tan(3m+1) + C$

4) $\int 2t\,(3t^2-1)^{1/2}\,dt$

$u = \sqrt{3t^2 - 1}$

$u^2 = 3t^2 - 1$

$3t^2 = u^2 + 1$

$t^2 = \frac{u^2+1}{3}$

$t = \sqrt{\frac{u^2+1}{3}}$

$\frac{dt}{du} = \frac{1}{2}\left(\frac{u^2+1}{3}\right)^{-1/2} \cdot \frac{2u}{3}$

$\frac{dt}{du} = \frac{u}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$$dt = \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$$

$$\int 2\left(\frac{u^2+1}{3}\right)^{1/2} \cdot u \cdot \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$$

$$= \frac{2}{3}\int u^2 \left(\frac{u^2+1}{3}\right)^{1/2-1/2} du$$

$$= \frac{2}{3}\int u^2\,du$$

$$= \frac{2}{3}\left[\frac{u^3}{3}\right] + C$$

$$= \frac{2u^3}{9} + C$$

$$= \frac{2(3t^2-1)^{3/2}}{9} + C$$

5 $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

$u = \sqrt{4x^2-1}$

$u^2 = 4x - 1$

$4x^2 = u^2 + 1$

$x^2 = \frac{u^2+1}{4}$

$x = \sqrt{\frac{u^2+1}{4}}$

$\frac{dx}{du} = \frac{1}{2}\left(\frac{u^2+1}{4}\right)^{-1/2} \cdot \frac{u}{2}$

$\frac{dx}{du} = \frac{u}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$

$$\int \frac{2\left(\frac{u^2+1}{4}\right)^{1/2} \cdot u\,du}{u} \cdot \frac{1}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$$

$$\frac{1}{2}\int \left(\frac{u^2+1}{4}\right)^{\frac{1}{2}-\frac{1}{2}} du$$

$$\frac{1}{2}\int du$$

$$= \frac{u}{2} + C$$

$$= \frac{\sqrt{4x^2-1}}{2} + C$$