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19/ENG04/036
Electrical and Electronics Engineering
MAT104

Differentiate the following

1) $y = [(x+1)^2(x-2)^{1/2}]/[(2x-1)(x-3)^{4/3}]$

$\ln y = \ln[(x+1)^2] + \ln[(x-2)^{1/2}] - \ln(2x-1) - \ln[(x-3)^{4/3}]$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{(x+1)^2} \quad 2x+2 + \frac{1}{(x-2)^{1/2}} \quad \frac{1}{2}(x-2)^{-1/2}$

$- \frac{1}{(2x-1)} \cdot (2) - \frac{1}{(x-3)^{4/3}} \cdot \frac{4}{3}(x-3)^{1/3}$

$\frac{1}{y}\frac{dy}{dx} = \frac{2x+2}{(x+1)^2} + \frac{\frac{1}{2}(x-2)^{-1/2}}{(x-2)^{1/2}} - \frac{2}{(2x-1)}$

$\frac{\frac{4}{3}(x-3)^{1/3}}{(x-3)^{4/3}}$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{(x+1)} + \frac{1}{2} - \frac{2}{(2x-1)} - \frac{\frac{4}{3}(x-3)^{1/3}}{(x-3)^{4/3}}$

$\frac{dy}{dx} = y\left[\frac{2}{(x+1)} + \frac{1}{2} - \frac{2}{(2x-1)} - \frac{\frac{4}{3}(x-3)^{1/3}}{(x-3)^{4/3}}\right]$

$\frac{dy}{dx} = \frac{(x+1)^2(x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}\left[\frac{2}{(x+1)} + \frac{1}{2} - \frac{2}{(2x-1)} - \frac{\frac{4}{3}(x-3)^{1/3}}{(x-3)^{4/3}}\right]$

2) $y = [3e^k \sin 2k]/k^{5/2}$

$\ln y = \ln(3e^k) + \ln(\sin 2k) - \ln(k^{5/2})$

$\frac{1}{y}\frac{dy}{\cancel{dk}} = \frac{1}{3e^k} \cdot 3e^k + \frac{1}{\sin 2k} \quad 2\cos 2k - \frac{1}{k^{5/2}} \quad 5k^{3/2}$

$\frac{dy}{dk} = 1 + \frac{2\cos 2k}{\sin 2k} - \frac{5k^{5/2}}{2k^{5/2}}$

$\frac{1}{y}\frac{dy}{dx} = 1 + 2\tan 2k - \frac{5k^{3/2}}{2k^{5/2}}$

$$\frac{dy}{dk} = y\left[1 + 2\tan 2k - \frac{5k^{3/2}}{2k^{3/2}}\right]$$

$$\therefore \frac{dy}{dk} = \frac{3e^{x}\sin 2k}{k^{5/2}}\left[1 + 2\tan 2k - \frac{5k^{5/2}}{2k^{5/2}}\right]$$

$\boxed{\text{Integration}}$

1) $4\sec^2(3m+1)$

let $u = 3m+1$

$\frac{du}{dm} = 3 \qquad dx = \frac{du}{3}$

$$= \int 4\sec^2 u \cdot \frac{du}{3} = \frac{1}{3}\int 4\sec^2 u \, du$$

$$= \frac{1}{3}\cdot\left[4\tan u + c\right] = \frac{4\tan u}{3} + c$$

$$\Rightarrow \frac{4\tan(3m+1)}{3} + c$$

2) $\int 2t(3t^2-1)^{1/2}\,dt$

let $u = 3t^2 - 1$

$du = 6t \qquad dt = \frac{du}{6t}$

$$\Rightarrow \int 2t\, u^{1/2}\,\frac{du}{6t} = \int u^{1/2}\frac{du}{3}$$

$$\frac{1}{3}\int u^{1/2}du \Rightarrow \frac{1}{3}\left[\frac{u^{3/2}}{3/2} + c\right]$$

$$= \frac{2u^{3/2}}{9} + c$$

$$\Rightarrow \frac{2(3t^2-1)^{3/2}}{9} + c$$

3) $\displaystyle\int \frac{2x}{(4x^2-1)^{1/2}}\,dx$

let $u = 4x^2 - 1$

$\dfrac{du}{dx} = 8x \qquad dx = \dfrac{du}{8x}$

$\Rightarrow \displaystyle\int \frac{2x}{u^{1/2}}\,dx = \int \frac{2x}{u^{1/2}}\,\frac{du}{8x}$

$= \dfrac{1}{4}\displaystyle\int u^{-1/2}\,du = \dfrac{1}{4}\int \frac{u^{1/2}}{1/2} + c$

$= \dfrac{u^{1/2}}{2} + c \Rightarrow \dfrac{(4x^2-1)^{1/2}}{2} + c$

OR $\dfrac{\sqrt{(4x^2-1)}}{2} + c$