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Level: 100

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1. Find the limit of the function $\{(x - \cos x)/x\}$ as $x \to 0$

Soln

$$\lim_{x \to 0} \frac{(x - \cos x)}{x}$$

Using L'hopitals rule,

$$\lim_{x \to 0} \frac{1 - (-\sin x)}{1}$$

$$= \lim_{x \to 0} \frac{(1 + \sin x)}{1} = \frac{1 + \sin 0}{1}$$

$$= \frac{1+0}{1} = 1$$

2. $y = -3\tan 7x \, e^{3x}$ dy/dx?

$u = -3\tan 7x$ $v = e^{3x}$

$\frac{du}{dx} = 7\sec^2(7x)$, $\frac{dv}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= (-3\tan 7x)(3e^{3x}) + (e^{3x})[7\sec^2(7x)]$$

$$= -9\tan 7x \, e^{3x} + 7\sec^2 7x \, e^{3x}$$

3. $y = \cos 3x$

$$\frac{dy}{dx} \lim_{x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{\cos[3(x+\Delta x)] - \cos 3x}{\Delta x}$$

$$= \frac{\cos(3x + 3\Delta x) - \cos 3x}{\Delta x}$$

Recall,

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$= \frac{[\cos 3x \cos(3\Delta x) - \sin 3x \sin(3\Delta x)] - \cos 3x}{\Delta x}$$

$$= \left[\frac{\cos 3x \cos(3\Delta x) - \cos 3x}{\Delta x}\right] - \left[\frac{\sin 3x \sin(3\Delta x)}{\Delta x}\right]$$

$$= \left[(\cos 3x)\left(\frac{\cos(3\Delta x) - 1}{\Delta x}\right)\right] - \left[(\sin 3x)\left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$$

$$= \left[(\cos 3x)\left(\frac{\cos^2(3\Delta x) - 1}{\cos(3\Delta x + 1)\Delta x}\right)\right] - \left[(\sin 3x)\left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2(3\Delta x) - 1 = -\sin^2(3\Delta x)$

$$= \left[(-\cos 3x)\left(\frac{\sin^2(3\Delta x)}{\cos(3\Delta x+1)\Delta x}\right) - (\sin 3x)\left(\frac{\sin(3\Delta x)}{\Delta x}\right) \right]$$

$$= \left[\frac{-\cos 3x \sin^2(3\Delta x) - \sin 3x}{\cos(3\Delta x+1)\Delta x} \right] \left[\frac{\sin(3\Delta x)}{\Delta x} \right]$$

$$= \lim_{x \to 0} \left[\frac{-\cos 3x \sin^2(3\Delta x) - \sin 3x}{\cos(3\Delta x+1)\Delta x} \right] \cdot \lim_{x \to 0} \left[\frac{\sin(3\Delta x)}{\Delta x} \right]$$

$$LHS = \cancel{-\sin 3x} - \frac{\cos(3x)[\sin(0)]}{\cos(0+1)\,0} - \sin 3x$$

$$= 0 - \sin 3x$$

$$= -\sin 3x$$

$$RHS = \lim_{x \to 0} \frac{\sin(3\Delta x)}{\Delta x} = 3$$

$\therefore$ the derivative $= -\sin 3x \cdot 3$

$= -3\sin 3x$

4. $(f-g)(5)$

$= (2x^3 - 7x + 3x)(5)$

$= 2x^3 - 4x(5)$

$= 2(5)^3 - 4(5)$

$= 250 - 20$

$= 230$

5.) $f \circ g(x)$

$= 4(2x+3)^2 + 2$

$= 4(4x^2 + 12x + 9) + 2$

$= 16x^2 + 48x + 36 + 2$

$= 16x^2 + 48x + 38$

7. $y = x^2 \cos x$

$u = x^2 \quad , \quad v = \cos x$

$\frac{du}{dx} = 2x, \quad \frac{dv}{dx} = -\sin x$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x^2(-\sin x) + \cos x(2x)$

$= -x^2 \sin x + 2x \cos x$

$= x(-x \sin x + 2\cos x)$

6. $x^2 + 2xy + y^2 = 1020$

$2x + 2\left[y + x\left(dy/dx\right)\right] + 2y\left(dy/dx\right) = 0$

$2x + 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$(2x + 2y) + \frac{dy}{dx}(2x + 2y) = 0$

$\frac{dy}{dx} = \frac{-(2x + 2y)}{(2x + 2y)}$

$\frac{dy}{dx} = -1$