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19/ENG08/037
MAT 104

(1) $\lim_{x \to 0} \left\{ \frac{x - \cos x}{x} \right\}$

Solution

$$\lim_{x \to 0} \left\{ \frac{x - \cos x}{x} \right\}$$

By direct substitution, we have $\frac{0}{0}$: Using L hopital's rule

$$\lim_{x \to} \left\{ \frac{x - \cos x}{x} \right\} = \lim_{x \to 0} \left\{ \frac{1 - (-\sin x)}{1} \right\}$$

$$= \lim_{x \to 0} \left\{ \frac{1 + \sin x}{1} \right\} = \frac{1 + \sin 0}{1} = \frac{1 + 0}{1}$$

$$= 1$$

(2) $y = -3 \tan 7x \, e^{3x}$

Solution

$y = -3 \tan 7x \, e^{3x}$

$u = -3 \quad v = \tan 7x \quad w = e^{3x}$

$du/dx = 0 \quad dv/dx = 7 \sec^2 7x \quad dw/dx = 3e^{3x}$

$$dy/dx = y \left[\frac{1}{u} \times \frac{du}{dx} + \frac{1}{v} \times \frac{dv}{dx} + \frac{1}{w} \times \frac{dw}{dx} \right]$$

$$= y \left[\frac{1}{3} \times 0 + \frac{1}{\tan 7x} \times 7 \sec^2 7x + \frac{1}{\cancel{e^{3x}}} \times 3\cancel{e^{3x}} \right]$$

$$= y \left[0 + \frac{7 \sec^2 7x}{\tan 7x} + 3 \right]$$

$$dy/dx = -3 \tan 7x \, e^{3x} \left[\frac{7 \sec^2 7x}{\tan 7x} + 3 \right]$$

(3) $y = \cos x$

Solution

$y = \cos x$

$y + \Delta y = \cos 3(x + \Delta x)$

$y + \Delta y = \cos 3x + 3\Delta x$

$\Delta y = \cos 3x + 3\Delta x - \cos 3x \quad \cdots (1)$

Recall $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2} \quad \cdots (2)$

Comparing (1) and (2)

$A = 3x + 3\Delta x \qquad B = 3x$

$\frac{A+B}{2} = \frac{3x + 3\Delta x - 3x}{2} = \frac{3\Delta x}{2}$

Hence $\Delta y = -2\sin\left(3x + \frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)$

divide through by Δx

$$\Delta y/\Delta x = \frac{-2\sin\left(3x + \frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)}{\Delta x}$$

Multiply both numerator and denominator through by $3/2$

$$\frac{-2\sin\left(3x + \frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right) \times 3/2}{\Delta x \times 3/2}$$

$$\Delta y/\Delta x = \frac{-3\sin\left(3x + \frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)}{3\Delta x/2}$$

$$\lim_{\Delta x \to 0} \Delta y/\Delta x = -3\sin 3x$$

$$\lim_{\to 0} x \frac{\sin\frac{3\Delta x}{2}}{\frac{3\Delta x}{2}} = 1$$

then $-3\sin 3x \times 1$

$\therefore dy/dx = -3\sin 3x$ //

(4) $f(x) = 2x^3 - 7x$

$g(x) = -3x$

$f(x) - g(x) = 2x^3 - 7x - (-3x)$

$= 2x^3 - 7x + 3x$

$= 2x^3 - 4x$

$(f - g)(5) = (2x^3 - 4x) \times 5$

$= 10x^3 - 20x$

dividing through by 10

$x^3 - 2x$ //

(5) $fog(x)$

$f(x) = 4x^2 + 2$

$g(x) = 2x + 3$

$fog(x) = 4(2x+3)^2 + 2$

$= 4(4x^2 + 12x + 9) + 2$

$= 16x^2 + 48x + 36 + 2$

$= 16x^2 + 48x + 38$

$fog(x) = 16x^2 + 48x + 38$

(6) $x^2 + 2xy + y^2 = 1020$

Solution

$x^2 + 2xy + y^2 = 1020$

$2x\frac{dx}{dx} + 2x\frac{dy}{dx} + 2y\frac{dx}{dx} + 2y\frac{dy}{dx} = 0$

$2x + 2x\,dy/dx + 2y + 2y\,dy/dx = 0$

$2x + 2y + dy/dx\,(2x + 2y) = 0$

$dy/dx\,(2x + 2y) = -2x - 2y$

$dy/dx = \frac{-2x - 2y}{2x + 2y}$ //

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(7) $y = x^2 \cos x$

Solution

Put $u = x^2$, $v = \cos x$

$du/dx = 2x$, $dv/dx = -\sin x$

Recall $u \cdot dv/dx + v \cdot du/dx$

$= x^2 - \sin x + \cos x \cdot 2x$

$-x^2 \sin x + 2x \cos x$

$= 2x \cos x - x^2 \sin x$ //