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MECHANICAL ENGINEERING
19/ENG06/016 SERIAL NO. ; 111
MAT 104 ASSIGNMENT (Mrs. Funmilayo Soko)

Differentiate the following;

1. $y = \frac{[(x+1)^2 (x-2)^{1/2}]}{[(2x-1)(x-3)^{4/3}]}$

2. $y = \frac{[3e^k \sin 2k]}{k^{5/2}}$

solution

1. $y = \frac{[(x+1)^2 (x-2)^{1/2}]}{[(2x-1)(x-3)^{4/3}]}$

$\ln y = [\ln(x+1)^2 + \ln(x-2)^{1/2}] - [\ln(2x-1) + \ln(x-3)^{4/3}]$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{(x+1)^2} \cdot 2(x+1) + \frac{1}{(x-2)^{1/2}} \cdot \frac{(x-2)^{-1/2}}{2} - \frac{1}{(2x-1)} \cdot 2 - \frac{1}{(x-3)^{4/3}} \cdot \frac{4(x-3)}{3}$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{2(x+1)}{(x+1)^2} + \frac{(x-2)^{-1/2}}{2(x-2)^{1/2}} - \frac{2}{(2x-1)} - \frac{4(x-3)^{1/3}}{3(x-3)^{4/3}}$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{2}{x+1} + \frac{1}{2(\sqrt{x-2})(\sqrt{x-2})} - \frac{2}{2x-1} - \frac{4(x-3)^{-1}}{3}$

$\frac{dy}{dx} = y\left[\frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{4}{3(x-3)}\right]$

$\frac{dy}{dx} = \frac{[(x+1)^2 (x-2)^{1/2}]}{[(2x-1)(x-3)^{\frac{4}{3}}]}\left[\frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{4}{3(x-3)}\right]$

2. $y = \frac{[3e^{x}\sin 2x]}{x^{5/2}}$

$\ln y = \ln(3e^{x}) + \ln(\sin 2x) - \ln(x^{5/2})$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{3e^{x}}\cdot 3e^{x} + \frac{1}{\sin 2x}\cdot 2\cos 2x - \frac{1}{x^{5/2}}\cdot\frac{5x^{3/2}}{2}$

$\frac{1}{y}\cdot\frac{dy}{dx} = 1 + \frac{2\cos 2x}{\sin 2x} - \frac{5x^{3/2}}{2x^{5/2}}$

$\frac{dy}{dx} = y\left[1 + \frac{2\cos 2x}{\sin 2x} - \frac{5x^{-1}}{2}\right]$

$\therefore \frac{dy}{dx} = \frac{[3e^{x}\sin 2x]}{x^{5/2}}\left[1 + \frac{2\cos 2x}{\sin 2x} - \frac{5}{2x}\right]$

- Integrate the following with respect to the variable;

1. $4\sec^2(3m+1)$
2. $2t(3t^2-1)^{1/2}$
3. $2x/(4x^2-1)^{1/2}$

Solution

1. ~~$4\sec^2$~~ $\int 4\sec^2(3m+1)\,dm$

$4\int \sec^2(3m+1)\,dm$

Let, $u = 3m+1$, $\frac{du}{dm} = 3$

$du = 3dm$

$dm = \frac{du}{3}$

$\Rightarrow 4\int \sec^2 u \cdot \frac{du}{3}$

$= \frac{4}{3}\int \sec^2 u \; du$

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$= \frac{4}{3}[\tan u] + C$

$= \frac{4 \tan u}{3} + C$

$= \frac{4 \tan(3m+1)}{3} + C$

2. ~~St~~ $\int 2t(3t^2-1)^{1/2}\, dt$

Let $u = (3t^2 - 1)^{1/2}$

$u = \sqrt{3t^2 - 1}$

$u^2 = 3t^2 - 1$

$3t^2 = u^2 + 1$

$t^2 = \frac{u^2+1}{3}$

$t = \sqrt{\frac{u^2+1}{3}}$

$t = \left(\frac{u^2+1}{3}\right)^{1/2}$

$p = \frac{u^2+1}{3}$ $\to w$ $\to x$

$\frac{dp}{du} = \frac{2u}{3}$, $t = p^{1/2}$

$\frac{dt}{dp} = \frac{1}{2} p^{-1/2}$

$$\therefore \frac{dt}{du} = \frac{2u}{3} \times \frac{1}{2} p^{-1/2}$$

$$= \frac{u}{3} p^{-1/2}$$

$$\frac{dt}{du} = \frac{u}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$$

$$3\,dt = u \cdot du \left(\frac{u^2+1}{3}\right)^{-1/2}$$

$$dt = \frac{u \cdot du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$$

$$\Rightarrow \int 2\left(\frac{u^2+1}{3}\right)^{1/2} \cdot u \cdot \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$$

$$= \frac{2}{3}\int \left(\frac{u^2+1}{3}\right)^{0} u^2 \cdot du$$

$$= \frac{2}{3}\left[\frac{u^{2+1}}{2+1}\right] + C$$

$$= \frac{2u^3}{9} + C$$

$$= \frac{2\left([3t^2-1]^{1/2}\right)^3}{9} + C$$

$$= \frac{2(3t^2-1)^{3/2}}{9} + C$$

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3. $\int \frac{2x}{(4x^2-1)^{1/2}}\,dx$

$\cancel{= \int 2x(4x^2-1)^{-1/2}\,dx}$

$\cancel{\text{let } u = (4x^2-1)^{-1/2}}$

Let,

$u = (4x^2-1)^{1/2}$

$u = \sqrt{4x^2-1}$

$u^2 = 4x^2-1$

$4x^2 = u^2+1$

$x^2 = \frac{u^2+1}{4}$

$x = \sqrt{\frac{u^2+1}{4}}$

$x = \left(\frac{u^2+1}{4}\right)^{1/2}$

$p = \frac{u^2+1}{4} \begin{matrix} \to w \\ \to x \end{matrix}$

$\frac{dp}{du} = \frac{u}{2}$; $x = p^{1/2}$

$\frac{dx}{dp} = \frac{1}{2}p^{-1/2}$

$\therefore \frac{dx}{du} = \frac{u}{2} \times \frac{1}{2}p^{-1/2}$

$= \frac{u}{4}p^{-1/2}$

$\frac{dx}{du} = \frac{u}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$

$$\cancel{4}dx = u \cdot du \left(\frac{u^2+1}{4}\right)^{-1/2}$$

$$dx = \frac{u \cdot du}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$$

$$\Rightarrow \int \frac{2\left(\frac{u^2+1}{4}\right)^{1/2}}{\cancel{u}} \cdot \frac{\cancel{u} \cdot du}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$$

$$= \int \frac{2\,du}{4}\left(\frac{u^2+1}{4}\right)^{0}$$

$$= \frac{1}{2}\int 1\,du$$

$$= \frac{1}{2}\left[\frac{u^{0+1}}{0+1}\right] + C$$

$$= \frac{u}{2} + C$$

$$= \frac{\sqrt{4x^2-1}}{2} + C$$

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