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DEPARTMENT: Computer Science

MATRIC NO: 19/SCIOI/039

1. Find the limit of the function $(x - \cos x)(x)$ as $x \rightarrow 0$

Solution

$$\lim_{x \rightarrow 0} (x - \cos x)(x)$$

By L'Hopital's rule

$$= \lim_{x \rightarrow 0} (1 - (-\sin x))$$

$$= \lim_{x \rightarrow 0} (1 + \sin x) = (1 + \sin(0)) = \frac{1+0}{1} = \frac{1}{1} = 1$$

2.) If $y = -3 \tan 7x e^{3x}$, find dy/dx

Solution

$$u = -3 \tan 7x \quad v = e^{3x}$$

$$\frac{du}{dx} = -21 \sec^2 7x \quad \frac{dv}{dx} = 3e^{3x}$$

$$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$\frac{dy}{dx} = (e^{3x})(-21 \sec^2 7x) + (-3 \tan 7x)(3e^{3x})$$

$$\frac{dy}{dx} = [(-21 \sec^2 7x)(e^{3x}) + e^{3x}(-3 \tan 7x)]$$

$$\frac{dy}{dx} = [e^{3x}(-21 \sec^2 7x) + e^{3x}(-3 \tan 7x)]$$

3.) If $y = \cos 3x$, find dy/dx from the first principle

Using the formula

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \frac{\cos [3(x-\Delta x)] - \cos 3x}{\Delta x}$$

$$= \frac{\cos [3x + 3\Delta x] - \cos 3x}{\Delta x}$$

Recall

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{[\cos 3x \cos(3\Delta x) - \sin 3x \sin(3\Delta x)] - \cos 3x}{\Delta x}$$

Collect like terms

$$= \left[\frac{\cos 3x \cos(3\Delta x) - \cos 3x}{\Delta x} \right] - \left[\frac{\sin 3x \sin(3\Delta x)}{\Delta x} \right]$$

$$= \lim_{\Delta x \rightarrow 0} \left[\cos 3x \left(\frac{\cos(3\Delta x) - 1}{\Delta x} \right) - \left[\sin 3x \left(\frac{\sin(3\Delta x)}{\Delta x} \right) \right] \right]$$

$$= \lim_{\Delta x \rightarrow 0} \left[\cos 3x \left(\frac{\cos(3\Delta x) - 1}{\Delta x} \times \frac{\cos(3\Delta x + 1)}{\cos(3\Delta x + 1)} \right) - \left[\sin 3x \left(\frac{\sin(3\Delta x)}{\Delta x} \right) \right] \right]$$

$$= \lim_{\Delta x \rightarrow 0} \left[\cos 3x \left(\frac{\cos^2(3\Delta x) - 1}{\cos(3\Delta x + 1) \Delta x} \right) - \left[\sin 3x \left(\frac{\sin 3\Delta x}{\Delta x} \right) \right] \right]$$

From the identity $\sin^2(\theta) + \cos^2(\theta) = 1$

$$\cos^2(3\Delta x) - 1 = -\sin^2(3\Delta x)$$

$$= \lim_{\Delta x \rightarrow 0} \left[-\cos 3x \left(\frac{\sin^2(3\Delta x)}{\cos(3\Delta x + 1) \Delta x} \right) - \left[\sin 3x \left(\frac{\sin(3\Delta x)}{\Delta x} \right) \right] \right]$$

$$= \lim_{\Delta x \rightarrow 0} \left[\frac{-\cos 3x \sin^2(3\Delta x) - \sin 3x}{\cos(3\Delta x + 1) \Delta x} \right] \left[\frac{\sin(3\Delta x)}{\Delta x} \right]$$

Since the limit of a product is the product of the limits, we have

$$= \left(\lim_{\Delta x \rightarrow 0} \left[\frac{-\cos 3x \sin^2(3\Delta x) - \sin 3x}{\cos(3\Delta x + 1) \Delta x} \right] \right) \cdot \left(\lim_{\Delta x \rightarrow 0} \left[\frac{\sin 3\Delta x}{\Delta x} \right] \right)$$

As $\Delta x \rightarrow 0$, the first limit goes to $-\frac{\cos 3x \sin^2(3x)}{\cos(0+1)x} - \sin 3x$,

which simplifies to $-\sin 3x$; so the derivative can be rewritten as $-\sin 3x \lim_{\Delta x \rightarrow 0} \frac{\sin(3\Delta x)}{\Delta x}$

$$\text{Recall } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1, \therefore \lim_{\Delta x \rightarrow 0} \frac{\sin(3\Delta x)}{\Delta x} = 3$$

So the final form of the derivative is $(-\sin 3x)(3)$
 $= -3 \sin 3x$

4) Given that $f(x) = 2x^3 - 7x$ and $g(x) = -3x$, find $(f-g)(5)$

Solution

$$(f-g)(5) = f(5) - g(5)$$

$$(f-g)(5) = [2(5)^3 - 7(5)] - [-3(5)]$$

$$(f-g)(5) = [2(125) - 35] - [-15]$$

$$(f-g)(5) = [250 - 35] + 15$$

$$(f-g)(5) = 215 + 15$$

$$(f-g)(5) = \underline{\underline{\frac{230}{2}}}$$

5) Find $f \circ g(x)$

$$f(x) = 4x^2 + 2$$

$$g(x) = 2x + 3$$

Solution

$$f \circ g(x) = f(2x+3) + 2$$

$$= 4(2x+3)(2x+3) + 2$$

$$= 4(4x^2 + 6x + 6x + 9) + 2$$

$$= 4(4x^2 + 12x + 9) + 2$$

$$= 16x^2 + 48x + 36 + 2$$

$$= 16x^2 + 48x + \underline{\underline{38}}$$

6) Find the gradient of $x^2 + 2xy + y^2 = 1020$

Solution

$$x^2 + 2xy + y^2 = 1020$$

$$2x \frac{dx}{dx} + 2x \frac{dy}{dx} + 2y \frac{dx}{dx} + 2y \frac{dy}{dx} = 0$$

$$2x + 2x \frac{dy}{dx} + 2y + 2y \frac{dy}{dx} = 0$$

$$= 2x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2x - 2y$$

$$\frac{dy}{dx} (2x + 2y) = -2x - 2y$$

$$\frac{dy}{dx} = \frac{-2x - 2y}{2x + 2y}$$

7.) Find the first derivative of the function $y = x^2 \cos x$

Solution

$$y = u^2 \cos u$$

$$u = x^2$$

$$\frac{du}{dx} = 2x$$

$$v = \cos x$$

$$\frac{dv}{dx} = -\sin x$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$= (x^2)(-\sin x) + (2x)(\cos x)$$

$$= -x^2 \sin x + 2x \cos x$$

$$= x(-x \sin x + 2 \cos x)$$

$$= x \cancel{(-x \sin x + 2 \cos x)} [2x \cos x - x^2 \sin x]$$

$f(x)$

$$f(x) = 4x^2 + 2$$

$$g(x) = 2x + 3$$

$$f \circ g(x) = 4(2x + 3)^2 + 2$$

$$= 4(2x + 3)(2x + 3) + 2$$

$$= 4(4x^2 + 5x + 6x + 9) + 2$$

$$= 4(4x^2 + 12x + 9) + 2$$

$$= 16x^2 + 48x + 36 + 2$$

$$= 16x^2 + 48x + 38$$