

AZANOR Micheal Ogheneta

19/ENG02/008

52

MAT 104

A) Differentiate

$$1) y = \frac{(x+1)^2 (x-2)^{\frac{1}{2}}}{(2x-1)(x-3)^{\frac{3}{2}}}$$

$$y = \left[\ln(x+1)^2 + \ln(x-2)^{\frac{1}{2}} \right] - \left[\ln(2x-1) + \ln(x-3)^{\frac{3}{2}} \right]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \left[\frac{1}{(x+1)^2} \cdot \frac{2(x+1)+1}{(x-2)^{\frac{1}{2}}} \cdot \frac{(x-2)^{-\frac{1}{2}}}{2} \right] - \left[\frac{1}{2x-1} \cdot \frac{2+1}{(x-3)^{\frac{3}{2}}} \cdot \frac{3(x-3)^{\frac{1}{2}}}{2} \right]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \left[\frac{2(x+1) + (x-2)^{\frac{1}{2}}}{(x+1)^2} \right] - \left[\frac{2}{2x-1} + \frac{3(x-3)^{\frac{1}{2}}}{2(x-3)^{\frac{3}{2}}} \right]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = y \left[\frac{2}{(x+1)} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{3}{2(x-3)^2} \right]$$

$$\frac{dy}{dx} = \frac{(x+1)^2 (x-2)^{\frac{1}{2}}}{(2x-1)(x-3)^{\frac{3}{2}}} \left[\frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{3}{2(x-3)^2} \right]$$

$$2) y = \frac{3e^{2x} \sin 2x}{x^{\frac{3}{2}}}$$

- Find the logs of both sides

$$\ln y = \ln 3e^{2x} + \ln \sin 2x - \ln x^{\frac{3}{2}}$$

Differentiate w/ the representation

$$\frac{d}{dx} (\ln y) = \frac{d}{dx} (3e^{2x}) + \frac{d}{dx} (\ln \sin 2x) - \frac{d}{dx} (\ln x^{\frac{3}{2}})$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{3e^{2x}} (3e^{2x}) + \frac{1}{\sin 2x} (\cos 2x) - \frac{1}{x^{\frac{3}{2}}} (3x^{\frac{1}{2}})$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{3e^x}{3e^x} + \frac{\cos 2x}{\sin 2x} - \frac{5/2 x^{3/2}}{x^{3/2}}$$

m.b.s by

$$\frac{1}{y} \frac{dy}{dx} \times y = y \left(\frac{3e^x}{3e^x} + \frac{\cos 2x}{\sin 2x} - \frac{5/2 x^{3/2}}{x^{3/2}} \right)$$

$$\frac{dy}{dx} = y \left(1 + \frac{\cos 2x}{\sin 2x} - \frac{5/2 x^{3/2}}{x^{3/2}} \right)$$

$$\frac{dy}{dx} = \frac{3e^x \sin 2x}{x^{3/2}} \left(1 + \frac{\cos 2x}{\sin 2x} - \frac{5/2 x^{3/2}}{x^{3/2}} \right)$$

B.) Integration

$$\int 4 \sec^2(3m+1) dm$$

$$u = 3m+1$$

$$du = 3dm$$

$$dm = \frac{du}{3}$$

$$\int 4 \sec^2 u \frac{du}{3}$$

$$\frac{4}{3} \int \sec^2 u du$$

Integration of $\sec^2 u = \tan u + C$

$$\frac{4}{3} \tan u + C$$

$$\frac{4}{3} \tan(3m+1) + C$$

$$2) \int 2t \times (3t^2 - 1)^{\frac{1}{2}}$$

$$u = 3t^2 - 1$$

$$\frac{du}{6t} = \frac{6t}{6t} dt$$

$$dt = \frac{du}{6t}$$

$$\int 2t \times (u)^{\frac{1}{2}} \frac{du}{6t}$$

$$\int \frac{1}{3} \times u^{\frac{1}{2}} du$$

$$\frac{1}{3} \int u^{\frac{1}{2}} du$$

$$= \frac{1}{3} \times \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{1}{3} \times \frac{2}{3} u^{\frac{3}{2}} + C$$

$$= \frac{2}{9} u^{\frac{3}{2}} + C$$

$$= \frac{2}{9} (3t^2 - 1)^{\frac{3}{2}} + C$$

$$3) \int \frac{2x}{(4x^2-1)^{1/2}} dx = \int 2x (4x^2-1)^{-1/2} dx$$

$$u = 4x^2 - 1$$

$$du = 8x dx$$

$$dx = \frac{du}{8x}$$

$$= \int 2x (u)^{-1/2} \frac{du}{8x}$$

$$= \frac{1}{4} \int u^{-1/2} du$$

$$= \frac{1}{4} \times \frac{u^{-1/2+1}}{-1/2+1} = \frac{1}{4} \times \frac{u^{-1/2+1}}{1/2+1}$$

$$= \frac{1}{4} \times \frac{u^{1/2}}{1/2}$$

$$= \frac{1}{4} \times 2u^{1/2}$$

$$= \frac{1}{2} u^{1/2} = \frac{1}{2} (4x^2-1)^{1/2}$$