SN 97

Aruwoserkpaye Oghenevwede Michail
19/ENG06/010
Mechanical Engineering

Maths 104

(1) $y = \dfrac{(x+1)^2(x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}$

$\ln y = \left[\ln\left((x+1)^2\right) + \ln\left((x-2)^{1/2}\right)\right] - \left[\ln(2x-1) + \ln\left((x-3)^{4/3}\right)\right]$

$\frac{1}{y}\frac{dy}{dx} = \left[\frac{1}{(x+1)^2} \cdot 2(x+1) + \frac{1}{(x-2)^{1/2}} \cdot \frac{(x-2)^{-1/2}}{2}\right] - \left[\frac{1}{(2x-1)} \cdot 2 + \frac{1}{(x-3)^{4/3}} \cdot \frac{4(x-3)^{1/3}}{3}\right]$

$\frac{1}{y}\frac{dy}{dx} = \left[\frac{2(x+1)}{(x+1)^2} + \frac{\frac{1}{\sqrt{x-2}}}{2\sqrt{x-2}}\right] - \left[\frac{2}{(2x-1)} + \frac{4(x-3)^{1/3}}{3(x-3)^{4/3}}\right]$

$\frac{1}{y}\frac{dy}{dx} = \left[\frac{2}{(x+1)} + \frac{1}{2(x-2)}\right] - \left[\frac{2}{2x-1} + \frac{4}{3(x-3)}\right]$

$\frac{dy}{dx} = y\left[\frac{2}{(x+1)} + \frac{1}{2(x-2)} - \frac{2}{(2x-1)} - \frac{4}{3(x-3)}\right]$

$\Rightarrow \frac{dy}{dx} = \frac{(x+1)^2(x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}\left[\frac{2}{(x+1)} + \frac{1}{(2x-4)} - \frac{2}{(2x-1)} - \frac{4}{3x-9}\right]$

(2) $y = \dfrac{3e^x \sin 2x}{k^{5/2}}$

$\ln y = \ln(3e^x) + \ln(\sin 2x) - \ln(k^{5/2})$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{3e^x} \cdot 3e^x + \frac{1}{\sin 2x} \cdot 2\cos 2x - \frac{1}{k^{5/2}} \cdot \frac{5k^{3/2}}{2}$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{3e^x}{3e^x} + \frac{2\cos 2x}{\sin 2x} - \frac{5k^{3/2 - 5/2}}{2k}$

$\frac{1}{y}\cdot\frac{dy}{dx} = \left[1 + \frac{2\cos 2x}{\sin 2x} - \frac{5}{2k}\right]$

$\frac{dy}{dx} = y\left[1 + \frac{2\cos 2x}{\sin 2x} - \frac{5}{2k}\right]$

$\Rightarrow \frac{dy}{dx} = \frac{3e^x \sin 2x}{k^{5/2}}\left[1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2k}\right]$

(3) $\int 4\sec^2(3m+1)$

Let $u = 3m+1 \therefore \frac{du}{dm} = 3$, $dm = \frac{1}{3}du$

$\int 4\sec^2 u\,dm = \frac{1}{3}\int 4\sec^2 u \cdot dm$

$= \frac{1}{3} \cdot 4\tan u + C$

$= \frac{4}{3}\tan u + C$

$\Rightarrow = \frac{4}{3}\tan(3m+1) + C$

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(4) $\int 2t(3t^2-1)^{1/2}$

Let $u = 3t^2-1$, $du/dt = \cancel{x}6t$ $\therefore$ $1/6\, du = t\,dt$

$$\int 2t \cdot u^{1/2}\,dt = \int 2u^{1/2}\, t\,dt$$
$$= \int 2u^{1/2}\left(1/6\, du\right)$$
$$= 1/6 \int 2u^{1/2}\, du$$
$$= \frac{1}{3}\int u^{1/2}\, du$$
$$= \frac{1}{3} \times \frac{2}{3} \times u^{3/2} + C$$
$$= 2/9\, u^{3/2} + C$$
$$\Longrightarrow \quad = 2/9\,(3t^2-1)^{3/2} + C$$

(5) $\dfrac{2x}{(4x^2-1)^{1/2}}\,dx$

Let $u = 4x^2-1$, $du/dx = 8x$ $\therefore$ $du/8 = x\,dx$

$$\int \frac{2}{\sqrt{4x^2-1}} \cdot \frac{du}{8} = 2\int \frac{1}{\sqrt{u}} \cdot \frac{du}{8}$$
$$= 2/8 \int \frac{1}{\sqrt{u}} \cdot du$$
$$= \frac{1}{4}\int \frac{1}{\sqrt{u}} \cdot du$$

Let $t = \sqrt{u}$

$$\frac{dt}{du} = \frac{1}{2\sqrt{u}}$$
$$2dt = \frac{1}{\sqrt{u}}\, du$$
$$\Rightarrow \frac{1}{4}\int 2dt = \frac{1}{2}\int dt$$
$$= \frac{1}{2}t + C$$
$$= \frac{1}{2}\sqrt{u} + C$$
$$\Longrightarrow \frac{1}{2}\sqrt{4x^2-1} + C$$