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MAT 104 ASSIGNMENT

① $\lim_{x \to 0} \frac{x - \cos x}{x}$

$\lim_{x \to 0} \left[\frac{x}{x} - \frac{\cos x}{x}\right]$

$\lim_{x \to 0} \left[1 - \frac{\cos x}{x}\right]$

$\lim_{x \to 0} 1 - \lim_{x \to 0} \left[\frac{\cos x}{x}\right]$

$\rightarrow \lim_{x \to 0} \left[\frac{\cos x}{x}\right] \simeq 0$

This is

$= 1 - 0 = 1$

② $y = -3 \tan 7x \, e^{3x}$

let $y = uv$

where $u = -3 \tan 7x$ $du/dx = -21 \sec^2 7x$

$v = e^{3x}$ $dv/dx = 3e^{3x}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= -3 \tan 7x (3e^{3x}) + e^{3x}(-21 \sec^2 7x)$

$= -9 \tan 7x \, e^{3x} + (-21 \sec^2 7x \, e^{3x})$

$= -9 \tan 7x \, e^{3x} - 21 \sec^2 7x \, e^{3x}$

③ If $y = \cos 3x$, find dy/dx from the first principle

$y = \cos 3x$

$y + \Delta y = \cos 3(x + \Delta x)$

$\Delta y = \cos 3(x + \Delta x) - y$

$\Delta y = \cos(3x + 3\Delta x) - \cos 3x \quad -(1)$

Recall $\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \quad -(r)$

$\therefore A = 3x + 3\Delta x$

$B = 3x$

$\frac{A+B}{2} = \frac{3x + 3\Delta x + 3x}{2} = \frac{6x + 3\Delta x}{2} = 3x + \frac{3\Delta x}{2}$

$\frac{A-B}{2} = \frac{3x + 3\Delta x - 3x}{2} = \frac{3\Delta x}{2}$

Hence

$\Delta y = -2 \sin\left(3x + \frac{3\Delta x}{2}\right) \sin\left(\frac{3\Delta x}{2}\right)$

Divide both sides by Δx

$\frac{\Delta y}{\Delta x} = \frac{-2 \sin\left(3x + \frac{3\Delta x}{2}\right) \sin\left(\frac{3\Delta x}{2}\right)}{\Delta x}$

Multiply numerator and denominator by $1/2$

$\frac{\Delta y}{\Delta x} = \frac{-2 \sin\left(3x + \frac{3\Delta x}{2}\right) \sin\left(\frac{3\Delta x}{2}\right) \times 1/2}{\Delta x \times 1/2}$

$\frac{\Delta y}{\Delta x} = \frac{-\sin\left(3x + \frac{3\Delta x}{2}\right) \sin\left(\frac{3\Delta x}{2}\right)}{\frac{\Delta x}{2}}$

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = -\sin 3x + 0 \quad \lim_{\Delta x \to 0} \frac{\sin \frac{3\Delta x}{2}}{\frac{\Delta x}{2}}$

Recall

$\lim \frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}} = 1$

$\therefore \frac{\Delta x}{\Delta y} = -\sin 3x + 0 \cdot 3 \times 1$

$\therefore \frac{dy}{dx} = -3 \sin 3x$

④ $f(x) = 2x^3 - 7x$

$g(x) = -3x$

find $(f-g)(5)$

$(f-g)(5) = f(5) - g(5)$

$f(5) = 2(5)^3 - 7(5)$

$= 2(125) - 7(5)$

$= 250 - 35$

$= 215$

$g(5) = -3(5)$

$= -15$

$(f-g)5 = 215 - (-15)$

$= 230$

⑤ $fog(x)$: $f(x) = 4x^2 + 2$

$g(x) = 2x + 3$

$fog(x) = 4(4x^2 + 6x + 6x + 9) + 2$

$= 4(4x^2 + 12x + 9) + 2$

$= 16x^2 + 48x + 36 + 2$

$= 16x^2 + 48x + 38$

6) $x^2 + 2xy + y^2 = 1020$

$x^2 + 2xy + y^2 - 1020 = 0$

$2x + 2y + 2x\left(\frac{\Delta y}{\Delta x}\right) + 2\left(\frac{\Delta y}{\Delta x}\right) = 0$

$2x + 2y + \frac{\Delta y}{\Delta x}(2x + 2) = 0$

$\frac{\Delta y}{\Delta x}(2x+2) = -2x - 2y$

$\frac{\Delta y}{\Delta x} = \frac{-2x - 2y}{2x + 2}$

7.) $y = x^2 \cos x$

let $y = uv$

where $u = x^2 \quad \frac{du}{dx} = 2x$

$v = \cos x \quad \frac{dv}{dx} = -\sin x$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x^2(-\sin x) + \cos x(2x)$

$= -x^2 \sin x + 2x \cos x$