Alasa Ahmed Kasim
19/sci01/14
MAT 104
Computer Science

@ find the limit of the function as x tends to zero

$$\lim_{x \to 0} \frac{(x - \cos x)}{x}$$

By L Hospital rule,

$$= \lim_{x \to 0} \frac{(1 - (-\sin x))}{1}$$

$$= \lim_{x \to 0} \frac{(1 + \sin x)}{1} = \frac{1 + \sin(0)}{1} = \frac{1+0}{1} = 1/1$$

$$= 1$$

If $y = -3\tan 7x e^{3x}$

let $u = -3\tan 7x$, $v = e^{3x}$

$du/dx = -21\sec^2 7x$ and $\frac{dv}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= -3\tan 7x(3e^{3x}) + e^{3x}(21\sec^2 7x)$$

$y = \cos 3x$

find dy/dx using first principle

Solution

$y = \cos 3x$

$y + dy = \cos(3x + dx)$

$Dy = \cos(3x + dx) - y$

$Dy = \cos(3x + dx) - \cos 3x$

Recall $\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$

$A = 3x + Dx$, $B = 3x$

$$\frac{A+B}{2} = \frac{3x + dx + 3x}{2} = \frac{6x + dx}{2} = 3x + \frac{Dx}{2}$$

$$\frac{A-B}{2} = \frac{3x + dx - 3x}{2} = \frac{Dx}{2}$$

$$\frac{Dy}{dx} = \frac{-2\sin\left(3x + \frac{Dx}{2}\right)\sin\left(\frac{Dx}{2}\right) \times \frac{1}{2}}{Dx \times \frac{1}{2}}$$

$$\frac{Dy}{dx} = -3\sin(3x+0) \cdot \lim_{dx \to 0} \sin\frac{\left(\frac{3dx}{2}\right)}{\left(\frac{3dx}{2}\right)}$$

$$\lim_{dx \to 0}$$

Since $\lim_{x \to 0} \sin\frac{\left(\frac{3dx}{2}\right)}{\left(\frac{3dx}{2}\right)} = 1$

$$\frac{Dy}{dx} = -3\sin 3x$$

4) Given that $f(x) = 2x^3 - 7x$ and $g(x) = -3x$ find $(f-g)(5)$

Solu.

$$(f-g)(5) = f(5) - g(5)$$

$$(f-g)(5) = [2(5)^3 - 7(5)][-3(5)]$$

$$(f-g)(5) = [2(125) - 35] - [-15]$$

$$= 250 - 35 + 15$$

$$= 230$$

$$\therefore (f-g)(5) = 230$$

5) find $fog(x)$ if $f(x) = 4x^2 + 2$ and $g(x) = 2x + 3$

Solution

$$f(g(x)) = 4[2x+3]^2 + 2$$

$$= 4[4x^2 + 12x + 9] + 2$$

$$= 16x^2 + 48x + 36] + 2$$

$$= 16x^2 + 48x + 38$$

$$f(g(x)) = 16x^2 + 48x + 38$$

6) find the gradient of $x^2 + 2xy + y^2 = 10x$

Sol:-

$$2x + 2x\,dy/dx + 2y + 2y\,dy/dx = 0$$

$$2x + 2x\,dy/dx + 2y + 2y\,dy/dx = 0$$

$$(2x\,dy/dx) + (2y\,dy/dx) = -2x - 2y$$

$$dy/dx\,(2x + 2y) = -2x - 2y$$

$$dy/dx = \frac{-2x - 2y}{2x + 2y}$$

$$\therefore dy/dx = \frac{-2x - 2y}{2x + 2y}$$

7) find the first derivative of the function $y = x^2 \cos x$

Sol:-

using product rule

$$dy/dx = U\frac{dv}{dx} + V\frac{du}{dx}$$

$U = x^2 \qquad V = \cos x$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = -\sin x$

$$dy/dx = x^2 - \sin x + 2x\cos x$$

$$\therefore x(-x\sin x + 2\cos)$$