(PART 1)

LECTURER: DR OYELAMI
NAME: STEPHEN JETHRO E.
DEPARTMENT: AERONAUTICAL ENGINEERING
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1.) Find the limit of the function $\frac{x - \cos x}{x}$ as $x \to 0$

solution

$$\lim_{x \to 0} \left[\frac{x - \cos x}{x}\right]$$

Using taylor's series

$$\lim_{x \to 0} \left[\frac{x - \cos x}{x}\right] = \lim_{x \to 0} \left[\frac{1}{x}\left(x - \left(1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right)\right)\right]$$

$$= \lim_{x \to 0} \left[\frac{1}{x}\left(x - 1 + \frac{x^2}{2} - \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots\right)\right]$$

$$= \lim_{x \to 0} \left[1 - \frac{1}{x} + \frac{x}{2} - \frac{x^3}{4!} + \frac{x^5}{6!} + \cdots\right]$$

$$= 1$$

$$\therefore \lim_{x \to 0} \left[\frac{x - \cos x}{x}\right] = 1$$

2) If $y = -3\tan 7x e^{3x}$, find dy/dx

Solution

$$y = -3\tan 7x e^{3x}$$

let $u = \tan 7x$; $du/dx = 7\sec^2 7x$

$v = e^{3x}$; $dv/dx = 3e^{3x}$

$$y = -3uv$$

$$\frac{dy}{dx} = -3\left(u\frac{dv}{dx} + v\frac{du}{dx}\right) =$$

$$\frac{dy}{dx} = -3\left[\tan 7x \cdot 3e^{3x} + e^{3x} \cdot 7\sec^2 7x\right]$$

$$= -3\left[\tan 7x e^{3x} + 7\sec^2 7x e^{3x}\right]$$

$$\therefore \frac{dy}{dx} = -3\left[3\tan 7x e^{3x} + 7\sec^2 7x e^{3x}\right)$$

(PART 3)

$(f-g)(5) = 2(5)^3 - 4(5)$

$= 2(125) - 20 = 250 - 20$

$\therefore (f-g)(5) = 230$

5) Find $fog(x)$ if $f(x) = 4x^2 + 2$ and $g(x) = 2x + 3$.

Solution

$f(x) = 4x^2 + 2$

$g(x) = 2x + 3$

$fog(x) = 4(2x+3)^2 + 2$

$= 4(4x^2) - 12x + 9) + 2$

$= 16x^2 + 48x + 36 + 2$

$\therefore fog(x) = 16x^2 + 48x + 38$

6) Find the gradient of $x^2 + 2xy + y^2 = 1,020$

Solution

$x^2 + 2x + y^2 = 1020$

Differentiating implicitly

$2x + 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$2x\frac{dy}{dx} + 2y\frac{dy}{dx} = -(2x + 2y)$

$\frac{dy}{dx}(2x + 2y) = -(2x + 2y)$

$\frac{dy}{dx} = \frac{-(2x + 2y}{(2x + 2y)}$

$\therefore \frac{dy}{dx} = -1$

Hence, gradient $= m = \frac{dy}{dx} = \cancel{1} = -1$

(PART 2)

3.) If $y = \cos 3x$, find dy/dx from first principle

Solution

$y = \cos 3x$

$y + \Delta y = \cos(3x + 3\Delta x)$

$\Delta y = \cos(3x + 3\Delta x) - \cos 3x$

But, $\cos(A+B) = \cos A \cos B - \sin A \sin B$

Comparing coefficients

$\therefore \Delta y = (\cos 3x \cos 3\Delta x - \sin 3x \sin 3\Delta x) - \cos 3x$

$\Delta y = \cos 3x \cos 3\Delta x - \sin 3x \sin 3\Delta x - \cos 3x$

$$\frac{\Delta y}{\Delta x} = \frac{\cos 3x(\cos 3\Delta x - 1) - \sin 3x \sin 3\Delta x}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{\cos 3x(\cos 3\Delta x - 1)}{\Delta x} - \frac{\sin 3x \sin 3\Delta x}{\Delta x}$$

As $\Delta x \to 0$, $\cos 3\Delta x \to 1$ and $\frac{\sin 3\Delta x}{\Delta x} \to 3$

$$\Rightarrow \frac{\Delta y}{\Delta x} = \frac{\cos 3x(\cancel{\cos 3\Delta x} - 1)}{0} - \sin 3x(3)$$

$$\therefore \frac{\Delta y}{\Delta x} = -3\sin 3x$$

$$\text{Hence, } \frac{dy}{dx} = \underline{-3\sin 3x}$$

4) Given that $f(x) = 2x^{3} - 7x$ and $g(x) = -3x$ find $(f-g)(5)$

Solution

$f(x) = 2x^3 - 7x$

$g(x) = -3x$

$(f-g) = (2x^3 - 7x) - (-3x)$

$= 2x^3 - 7x + 3x$

$(f-g) = 2x^3 - 4x$

(PART 4)

7) find the first derivative of the function $y = x^2 \cos x$

Solution

$y = x^2 \cos x$

let $u = x^2$; $\frac{du}{dx} = 2x$

$v = \cos x$; $\frac{dv}{dx} = -\sin x$

$y = uv$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= x^2(-\sin x) + \cos x(2x)$$

$$\therefore \frac{dy}{dx} = -x^2 \sin x + 2x \cos x$$