

SOTANDE REBECCA

CIVIL ENGINEERING

19/ENG031026

MAT 102

Answers

$$A = 8\hat{i} - \hat{j}, B = 3\hat{i} + \hat{j} - 11\hat{k} \text{ and } C = 4\hat{i} + \hat{j} - 5\hat{k}$$

$$\begin{aligned} \textcircled{1} \quad -3A + 7B - 8C &= -3(8\hat{i} - \hat{j}) + 7(3\hat{i} + \hat{j} - 11\hat{k}) - 8(4\hat{i} + \hat{j} - 5\hat{k}) \\ &= -24\hat{i} + 3\hat{j} + 21\hat{i} + 7\hat{j} - 77\hat{k} - 32\hat{i} - 8\hat{j} + 40\hat{k} \\ &= (-6\hat{i} + 2\hat{j} - 32\hat{k}) + (3\hat{j} + 7\hat{j} - 32\hat{k}) + (-77\hat{k} + 40\hat{k}) \\ &= -6\hat{i} + 5\hat{j} - 24\hat{k} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad k &= 2A + 4B - C = 2(8\hat{i} - \hat{j}) + 4(3\hat{i} + \hat{j} - 11\hat{k}) - (4\hat{i} + \hat{j} - 5\hat{k}) \\ &= 16\hat{i} - 2\hat{j} + 12\hat{i} + 4\hat{j} - 44\hat{k} - 4\hat{i} - \hat{j} + 5\hat{k} \\ &= (16+12-4)\hat{i} + (-2+4-1)\hat{j} + (-44+5)\hat{k} \\ &= 24\hat{i} + 1\hat{j} - 39\hat{k} \end{aligned}$$

(iii) $C(A \times B \times C)$

$$(A \times B) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -1 & 0 \\ 3 & 1 & -11 \end{vmatrix}$$

$$\hat{i} \begin{vmatrix} -1 & 0 \\ 1 & -11 \end{vmatrix} - \hat{j} \begin{vmatrix} 8 & 0 \\ 3 & -11 \end{vmatrix} + \hat{k} \begin{vmatrix} 8 & -1 \\ 3 & 1 \end{vmatrix}$$

$$\hat{i}((-1)(-11) - 0) - \hat{j}(8(-11) - 0) + \hat{k}(8(1) - (-3))$$

$$11\hat{i} + 88\hat{j} + 11\hat{k}$$

$$C(A \times B \times C) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 11 & 88 & 11 \\ 4 & 4 & -5 \end{vmatrix}$$

$$\hat{i} \begin{vmatrix} 88 & 11 \\ 4 & -5 \end{vmatrix} - \hat{j} \begin{vmatrix} 11 & 11 \\ 4 & -5 \end{vmatrix} + \hat{k} \begin{vmatrix} 11 & 88 \\ 4 & 4 \end{vmatrix}$$

$$i(-110-24) - j(-55-24) + k(44-88) \\ = -134i + 79j - 44k$$

$$(iv) (3A \times B) \cdot (A \times 2B)$$

$$(3A \times B) = 3(2i - j) \times (3i + j - 11k) \\ (6i - 3j) \times (3i - j - 11k)$$

$$(3A \times B) = \begin{vmatrix} i & j & k \\ 6 & -3 & 0 \\ 3 & -1 & -11 \end{vmatrix} = i \begin{vmatrix} -3 & 0 \\ -1 & -11 \end{vmatrix} - j \begin{vmatrix} 6 & 0 \\ 3 & -11 \end{vmatrix} - k \begin{vmatrix} 6 & -3 \\ 3 & -1 \end{vmatrix}$$

$$i(-33+0) - j(-66-0) - k(-6+9)$$

$$= -33i + 66j - 3k$$

$$(3A \times 2B) = (2i + j) \times 2(2i + j - 11k)$$

$$2(2i - j) \times (6i + 2j - 22k)$$

$$(A \times 2B) = \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 6 & 2 & -22 \end{vmatrix} = i \begin{vmatrix} -1 & 0 \\ 2 & -22 \end{vmatrix} - j \begin{vmatrix} 2 & 0 \\ 6 & -22 \end{vmatrix} + k \begin{vmatrix} 2 & -1 \\ 6 & 2 \end{vmatrix}$$

$$= i(22-0) - j(-44+0) + k(4+6)$$

$$= 22i + 44j + 10k$$

$$(3A \times B) \cdot (A \times 2B) = (-33i + 66j - 3k) \cdot (22i + 44j + 10k)$$

$$= -66i + 2904j - 30k$$

$$\vee A - 2B - C = 2i - j(-2(2i + j - 11k)) - (4i + 4j - 5k)$$

$$A - 2B - C = 2i - j(-6i - 2j + 22k - 4i - 4j + 5k) \\ = 8i - 7j + 27k$$

2) Perpendicular vector: Two vectors A and B are said to be perpendicular

$$\text{if } A \cdot B = 0$$

iii) Co-planar vector: Three vectors A, B and C are said to be coplanar

$$\text{if } A \cdot (B \times C) = 0$$