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MAT 104

① $y = \frac{(x+1)^2 (x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}$

$\ln y = \ln[(x+1)^2] + \ln[(x-2)^{1/2}] - \ln[(2x-1)] - \ln[(x-3)^{4/3}]$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{(x+1)^2}\cdot(2x+2) + \frac{1}{(x-2)^{1/2}}\cdot\frac{1}{2}(x-2)^{-1/2} - \frac{1}{2x-1}\cdot(2) - \frac{1}{(x-3)^{4/3}}\cdot\frac{1}{4}(x-3)$

$\frac{1}{y}\frac{dy}{dx} = \frac{2x+2}{(x+1)^2} + \frac{1/2(x-2)^{-1/2}}{(x-2)^{1/2}} - \frac{2}{(2x-1)} - \frac{4/3(x-3)^{1/3}}{(x-3)^{4/3}}$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{(x+1)} + \frac{1}{2} - \frac{2}{2x-1} - \frac{4/3(x-3)^{1/3}}{(x-3)^{4/3}}$

$\frac{dy}{dx} = y\left[\frac{2}{(x+1)} + \frac{1}{2} - \frac{2}{(2x-1)} - \frac{4/3(x-3)^{1/3}}{(x-3)^{4/3}}\right]$

$\therefore \frac{dy}{dx} = \frac{(x+1)^2(x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}\left[\frac{2}{(x+1)} + \frac{1}{2} - \frac{2}{(2x-1)} - \frac{4/3(x-3)^{1/3}}{(x-3)^{4/3}}\right]$

2. $y = 3e^k \sin 2k$

$\ln y = \ln 3e^k + \ln(\sin 2k) - \ln(k^{5/2})$

$\frac{1}{y}\cdot\frac{dy}{dk} = \frac{1}{3e^k}\cdot 3e^k + \frac{1}{\sin 2k}\cdot 2\cos 2k - \frac{1}{k^{5/2}}\cdot 5/2\, k^{3/2}$

$\frac{1}{y}\cdot\frac{dy}{dk} = 1 + \frac{2\cos 2k}{\sin 2k} - \frac{5k^{3/2}}{2k^{5/2}}$

$\frac{1}{y}\cdot\frac{dy}{dk} = 1 + 2\tan 2k - \frac{5k^{3/2}}{2k^{5/2}}$

$$\frac{dy}{dk} = y\left[\frac{1 + 2\tan 2k - 5k^{5/2}}{2k^{5/2}}\right]$$

$$\therefore \frac{dy}{dk} = \frac{3e^{x}\sin 2k}{k^{5/2}}\left[\frac{1 + 2\tan 2k - 5k^{5/2}}{2k^{5/2}}\right]$$

Integration

1. $4\sec^2(3m+1) = y$

$$\int 4\sec^2(3m+1)$$

let $u = 3m+1$

$$\frac{du}{dm} = 3 \quad \therefore dx = \frac{du}{3}$$

$$= \int 4\sec^2 y \cdot \frac{du}{3} = \frac{1}{3}\cdot \int 4\sec^2 u\, du$$

$$\Rightarrow \frac{1}{3}[4\tan u + c] = 4\tan u + c$$

$$= \frac{4\tan(3m+1)}{3} + c$$

② $2t(3t^2-1)^{1/2}$

$$\int 2t(3t^2-1)^{1/2} dt$$

let $u = 3t^2 - 1$

$$\frac{du}{dt} = 6t \quad \therefore dt = \frac{du}{dt}$$

$$\Rightarrow \int 2t\, u^{1/2} \cdot \frac{du}{6t} = \int u^{1/2} \cdot \frac{du}{3}$$

$$\frac{1}{3}\int u^{1/2} du \Rightarrow \frac{1}{3}\left[\frac{u^{3/2}}{3/2} + c\right] \Rightarrow \frac{2u^{3/2}}{9} + c$$

③ $\frac{2x}{(4x^2-1)^{1/2}}$

$$\int \frac{2x}{(4x^2-1)^{1/2}} dx$$

let $u = 4x^2 - 1$

$$\frac{du}{dx} = 8x \quad \therefore dx = \frac{du}{dx}$$

$$\Rightarrow \int \frac{2x}{u^{1/2}} dx = \int \frac{2x}{u^{1/2}} \cdot \frac{du}{8x}$$

$$= \frac{1}{4}\int u^{-1/2} du = \frac{1}{4}\left[\frac{u^{1/2}}{1/2} + c\right]$$

$$= \frac{u^{1/2}}{2} + c \Rightarrow (4x^2-1)^{1/2} + c$$

or $\frac{\sqrt{4x^2-1}}{2} + c$