

Assignment:

Examine whether or not these pair of lines are perpendicular to each other.

1)  $y - 3x - 2 = 0$  and  $3y + x + 9 = 0$

Comparing the equations to equation of line  
 equation of line  $y = m_1x + c_1$

Equation 1,  $y - 3x - 2 = 0$

$y = 3x + 2$

$m_1 = 3$

Equation 2,  $3y + x + 9 = 0$

$3y = -x - 9$

$y = -\frac{x}{3} - 3$

$m_2 = -\frac{1}{3}$

Recall that perpendicular lines,  $m_1 m_2 = -1$

$3 \times -\frac{1}{3} = -1$

$\therefore y - 3x - 2 = 0$  and  $3y + x + 9 = 0$  are perpendicular to each other.

2)  $3y - 4 = 2x + 3$  and  $y - 5 = x + 6$

Comparing the equations to equation of line  
 Equation of line,  $y = m_1x + c_1$

Equation 1,  $3y - 4 = 2x + 3$

$3y = 2x + 7$

$y = \frac{2}{3}x + \frac{7}{3}$

$m_1 = \frac{2}{3}$

Equation 2,  $y - 5 = x + 6$

$y = x + 11$

$m_2 = 1$

Perpendicular line,  $m_1 m_2 = -1$

$\frac{2}{3} \times 1 = \frac{2}{3}$ ;  $\frac{2}{3} \neq -1$

$\therefore 3y - 4 = 2x + 3$  and  $y - 5 = x + 6$  are not perpendicular to each other.

3) Find the equations of the tangent and normal to the curve  $x^2 + y^2 + 3xy - 11 = 0$  at the point  $x=1, y=2$ .

Solution

$$m = \frac{dy}{dx} = \frac{2x + 2y \frac{dy}{dx} + 3[2x \frac{dy}{dx} + y(1)]}{2x + 2y \frac{dy}{dx} + 3[2x \frac{dy}{dx} + y(1)]} = 0$$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$2y \frac{dy}{dx} + 3x \frac{dy}{dx} = -3y - 2x$$

$$\frac{dy}{dx} (2y + 3x) = -(3y + 2x)$$

$$\frac{dy}{dx} = -\frac{(3y + 2x)}{(2y + 3x)}$$

$$\frac{dy}{dx} = -\frac{(3(2) + 2(1))}{2(2) + 3(1)} = -\frac{(6+2)}{4+3} = -\frac{8}{7}$$

Equation of the tangent;  $y - y_1 = m(x - x_1)$

$$y - 2 = -\frac{8}{7}(x - 1)$$

$$7y - 14 = -8x + 8$$

$$7y + 8x - 14 - 8 = 0$$

$7y + 8x - 22 = 0$ ; which is the equation of tangent.

Equation of normal;  $y - y_1 = -\frac{1}{m}(x - x_1)$

$$y - 2 = -\left(-\frac{7}{8}\right)(x - 1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$8y - 16 = 7x - 7$$

$$8y - 16 - 7x + 7 = 0$$

$8y - 7x - 9 = 0$ ; which is the equation of normal.